

Lecture 8

CFD for Ramjets/Scramjets and Rockets

- High compressibility in the flow
 - Shock-vortex-turbulence-flame interactions
- Challenge
 - Shock capturing schemes are too dissipative and can overwhelm turbulent features
 - Turbulent features must show compressibility effects
- Some strategies
 - Hybrid solvers (WENO-central; MUSCL-central etc)
 - Artificial dissipation, high order PPM etc
- What will work for practical applications?

Challenges for Supersonic Combustion LES

- Algorithms for shock-turbulence-flame interactions
 - Shock capturing without dissipating turbulence or affecting combustion or flame within LES framework
- Subgrid closure for compressible turbulent flows
 - Shock interactions with {un}resolved turbulence
- Subgrid closure for compressible mixing and combustion
 - Interaction of compressible waves with flames
- Molecular mixing and chemical kinetics within subgrid
 - Detailed kinetics within LES framework

Compressible LES Governing Equations

- Favre-averaged filtered conservation equations

Mass
$$\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial \bar{\rho} \tilde{u}_i}{\partial x_i} = 0$$

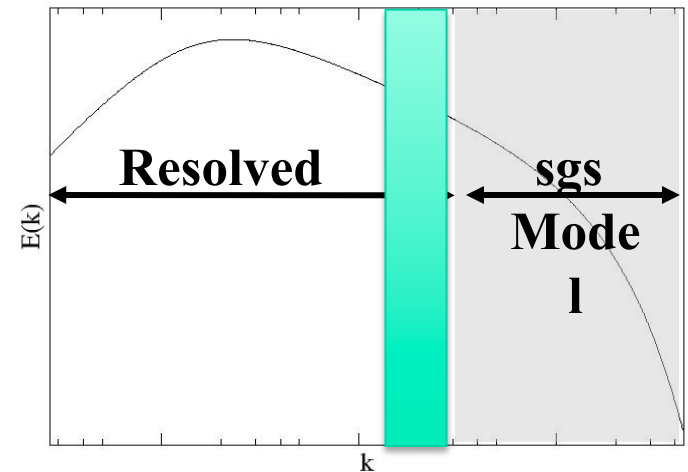
Moment.
$$\frac{\partial \bar{\rho} \tilde{u}_i}{\partial t} + \frac{\partial}{\partial x_j} \left(\bar{\rho} \tilde{u}_i \tilde{u}_j + \bar{p} \delta_{ij} - \bar{\tau}_{ij} + \tau_{ij}^{sgs} \right) = 0$$

Energy
$$\frac{\partial \bar{\rho} \tilde{E}}{\partial t} + \frac{\partial}{\partial x_i} \left[\tilde{u}_i (\bar{\rho} \tilde{E} + \bar{p}) - \bar{\tau}_{ij} \tilde{u}_j + \bar{q}_i + H_i^{sgs} + \sigma_i^{sgs} \right] = 0$$

E.O.S
$$\bar{p} = \bar{\rho} \tilde{R} \tilde{T} + \bar{\rho} T^{sgs}$$

$$\bar{q}_i = -\bar{K} \frac{\partial \tilde{T}}{\partial x_i} + \bar{\rho} \sum_{k=1}^{N_s} \tilde{h}_k \tilde{V}_{i,k} \tilde{Y}_k + \sum_{k=1}^{N_s} q_{i,k}^{sgs} \quad k = 1, N_s$$

- Species solved using LEMLES



Hybrid Algorithm for Shock-Turbulence-Flame

- Locally adaptive hybrid strategy switches from shock capturing solver to a smooth-flow ($O(4)$) solver dynamically in 3D
- Piecewise Parabolized Method (PPM – FLASH3D type)*
 - Extended to viscous flows, multi-domain, stretched grids
- MUSCL reconstruction with a Hybrid HLL Riemann Solver**
 - Non-contact preserving in shock transverse directions (Einfeldt, 1988, 1991)
 - Contact preserving Riemann solver (HLLC, Toro, 1997)
- Local shock detection using multiple sensors
- Algorithm validated for many canonical and complex test cases: Sod, Noh, Richtmyer-Meshkov, Shock-turbulence etc.**

VLES-LES k - k_l Model

- Hybrid RANS-LES for compressible flows using an additive filter (J. Comp. Phys. Vol. 228, 2009)
 - Hybrid terms need to be modeled – still under work
- Solve for the single point and two-point velocity correlations (k , k_l) for near-wall treatment – model is still under development
 - $l_{sgs} > \Delta$, the grid size is the length scale
 - $l_{sgs} < \Delta$, the modeled length scale is used
- Distance from wall is used currently in the isolator (K-DES)

$$\frac{\partial(\bar{\rho}k_{sgs})}{\partial t} + \frac{\partial(\bar{\rho}\tilde{u}_i k_{sgs})}{\partial x_j} = \tau_{ij} \frac{\partial \tilde{u}_i}{\partial x_j} - D_{sgs} + \frac{\partial}{\partial x_i} \left[\bar{\rho} \left(\frac{\nu}{Pr_t} + \frac{\nu_t}{\sigma_k} \right) \frac{\partial k_{sgs}}{\partial x_i} \right]$$

$$\frac{\partial(\bar{\rho}(k\ell)_{sgs})}{\partial t} + \frac{\partial(\bar{\rho}\tilde{u}_i (k\ell)_{sgs})}{\partial x_j} = C_{L1} \ell_{sgs} \tau_{ij} \frac{\partial \tilde{u}_i}{\partial x_j} - C_{L2} \bar{\rho} k_{sgs}^{3/2} + \frac{\partial}{\partial x_i} \left[\bar{\rho} \left(\frac{\nu}{Pr_t} + \frac{\nu_t}{\sigma_{k\ell}} \right) \frac{\partial (k\ell)_{sgs}}{\partial x_i} \right]$$

General Realizability Constraints

- Constraints on subgrid models (Vreman, *et al.*, 1994; Nelson and Menon, 1998, Fang and Menon, 2006) :

$$\tau_{\alpha\alpha}^{sgs} \geq 0 \quad \left(\tau_{\alpha\beta}^{sgs}\right)^2 \leq \tau_{\alpha\alpha}^{sgs} \tau_{\beta\beta}^{sgs} \quad \text{for } \alpha \neq \beta \quad \det\left(\tau_{\alpha\beta}^{sgs}\right) \geq 0$$

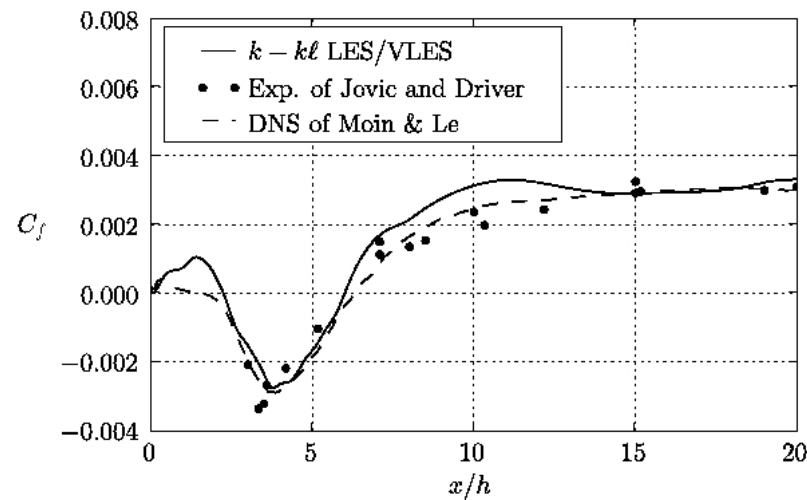
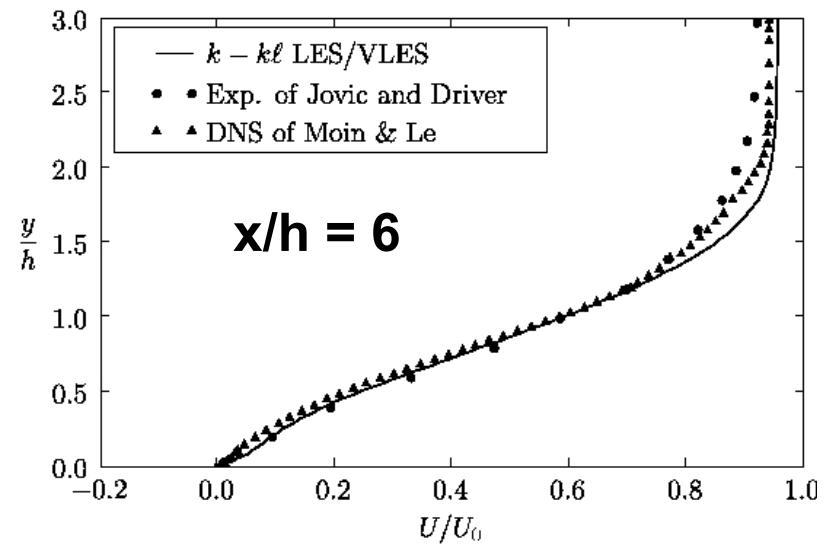
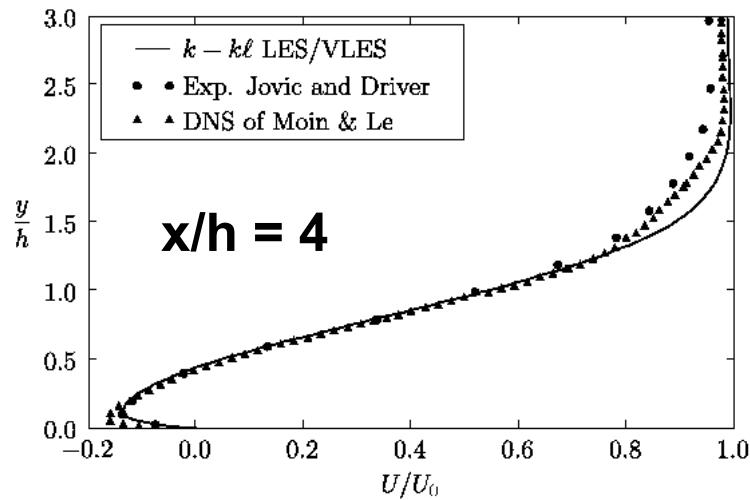
$$k^{sgs} \geq \frac{\sqrt{3}}{C_\alpha} \nu_t \sqrt{2\tilde{S}_{ij}\tilde{S}_{ji} - \frac{2}{3}\tilde{S}_{kk}^2} \quad \text{with} \quad \tilde{S}_{ij}\tilde{S}_{ji} = \sum_{i=1}^3 \sum_{j=1}^3 \tilde{S}_{ij}\tilde{S}_{ji}$$

$$C_\nu = \min(C_\nu, C_{\nu,\text{lim}}) \quad C_{\nu,\text{lim}} = \frac{1}{\sqrt{6}s}$$

$$s = \frac{l^{sgs}}{\sqrt{k^{sgs}}} \sqrt{\tilde{S}_{ij}\tilde{S}_{ji} - \frac{1}{3}\tilde{S}_{kk}^2}$$

$$l^{sgs} = \min(l^{sgs}, l_{\text{lim}}^{sgs}) \quad l_{\text{lim}}^{sgs} = \frac{1}{\sqrt{6}} \frac{\sqrt{k^{sgs}}}{C_\nu} \left(\tilde{S}_{ij}\tilde{S}_{ji} - \frac{1}{3}\tilde{S}_{kk}^2 \right)^{-0.5}$$

K-KL Rearward Facing Step



Scalar Fluctuation Modeling

- Used in typical RANS, URANS codes (e.g. CRAFTTech)
- Specify turbulent Prandtl and Schmidt numbers
 - First order effect impacts combustion efficiency
 - Use local estimates of turbulent Pr and Sc
 - **Adjust to the flow rather than set a priori**
 - **Obtained from the turbulent closure**
 - **Used in URANS and in conventional LES**
- **Dynamic subgrid closures avoid this explicit relations but also capture variable and local turbulent Pr and Sc.**

SCALAR FLUCTUATION MODEL (SFM)

Transport equations solved for scalar variance and its dissipation rate

$$\frac{\partial(\bar{\rho}k_e)}{\partial t} + \frac{\partial(\bar{\rho}\tilde{u}_j k_e)}{\partial x_j} = \frac{\partial}{\partial x_j} \left[\bar{\rho} \left(\alpha + \frac{\alpha_t}{\sigma_{k,e}} \right) \frac{\partial k_e}{\partial x_j} \right] + 2\bar{\rho}\alpha_t \left(\frac{\partial \tilde{e}}{\partial x_j} \right)^2 - 2\bar{\rho}\varepsilon_e$$

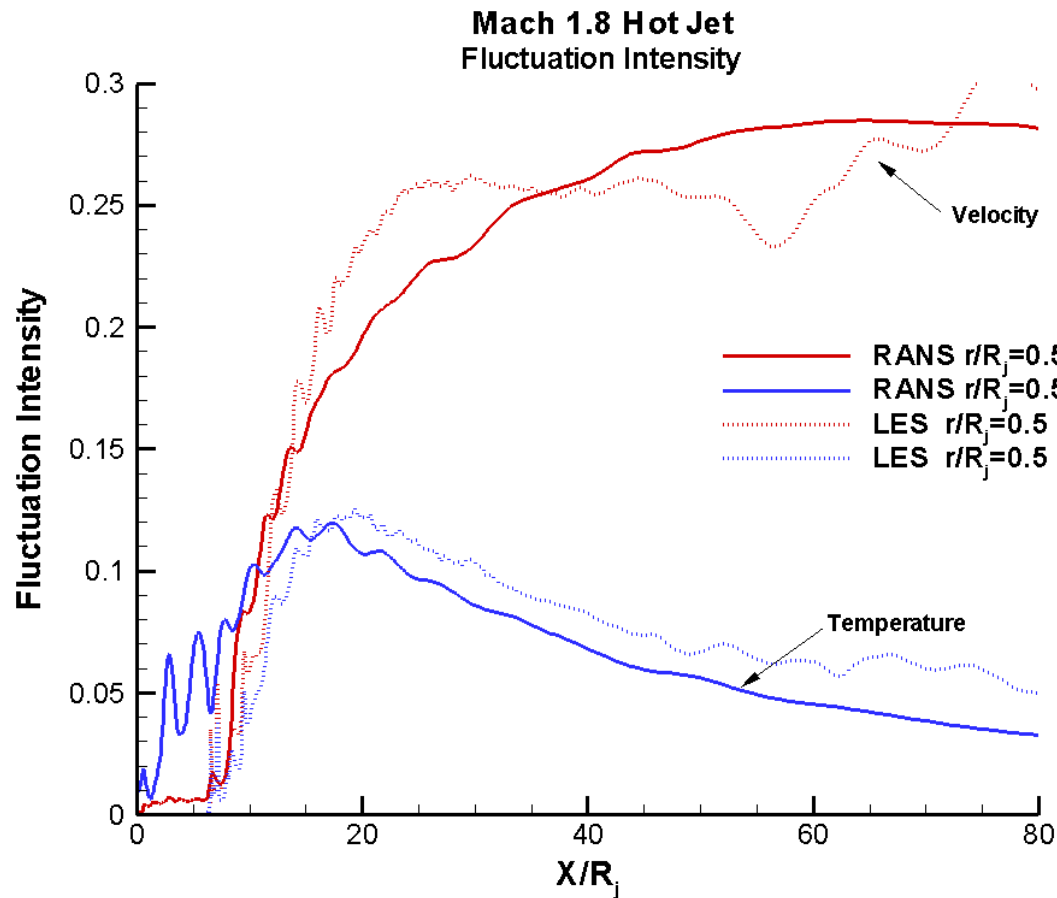
$$\begin{aligned} \frac{\partial(\bar{\rho}\varepsilon_e)}{\partial t} + \frac{\partial(\bar{\rho}\tilde{u}_j \varepsilon_e)}{\partial x_j} &= \frac{\partial}{\partial x_j} \left[\bar{\rho} \left(\alpha + \frac{\alpha_t}{\sigma_{\varepsilon,e}} \right) \frac{\partial \varepsilon_e}{\partial x_j} \right] + \bar{\rho}\alpha_t \left(C_{d1} \frac{\varepsilon_e}{k_e} + C_{d2} \frac{\varepsilon}{k} \right) \left(\frac{\partial \tilde{e}}{\partial x_j} \right)^2 \\ &+ C_{d3} \hat{P}_k \frac{\varepsilon_e}{k} - \left(C_{d4} \frac{\varepsilon_e}{k_e} + C_{d5} \frac{\varepsilon}{k} \right) \bar{\rho}\varepsilon_e + \xi_{\varepsilon T} \end{aligned}$$

Near-wall damping

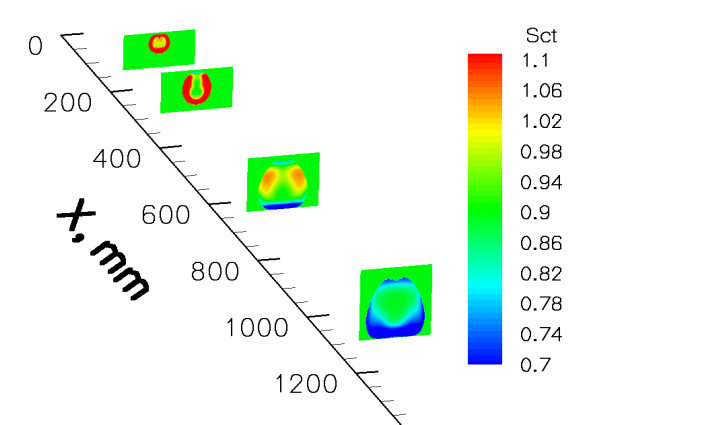
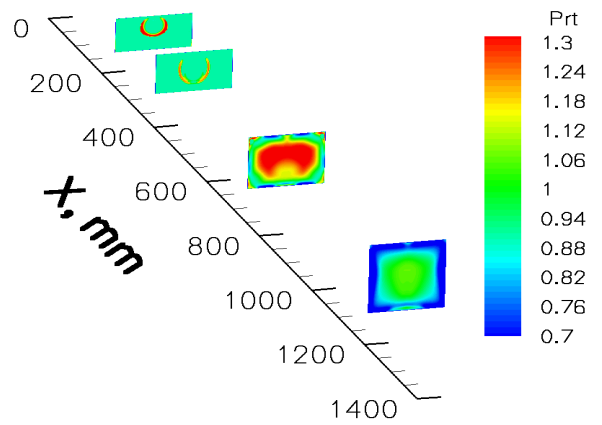
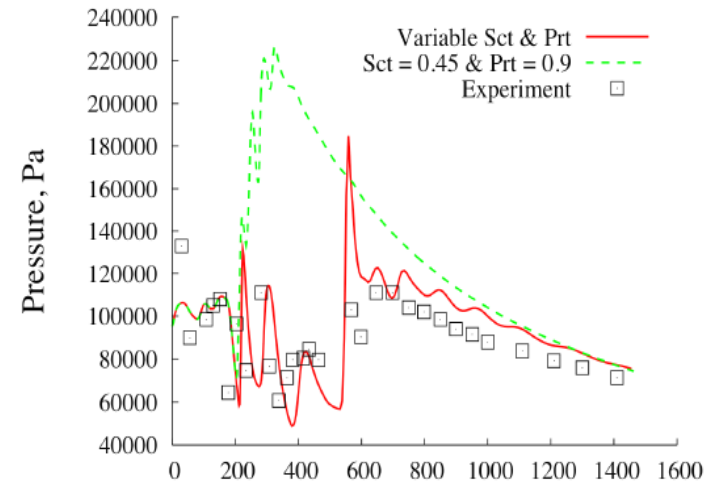
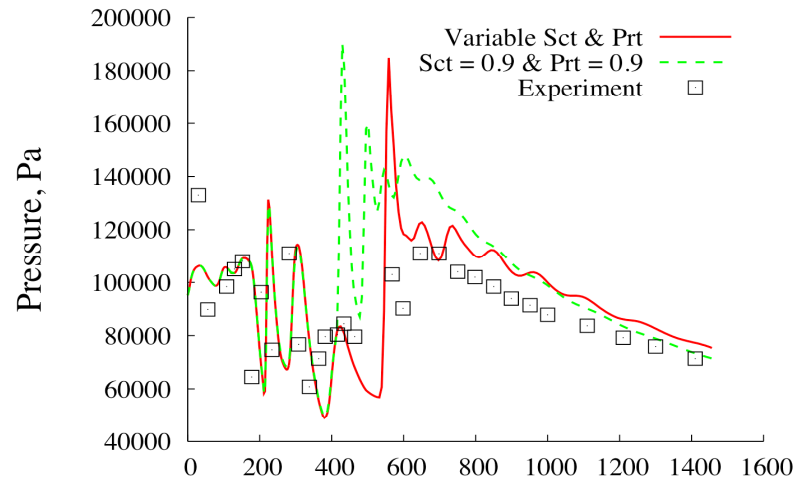
Compressibility Correction $\longrightarrow \hat{P}_k = P_k - \alpha_1 \hat{M}_T^2 P_k - \alpha_2 \hat{M}_T^2 \bar{\rho}\varepsilon$

Turbulent Prandtl Number			Turbulent Schmidt Number			
$Pr_t = \frac{C_\mu f_\mu}{C_\lambda f_\lambda} \sqrt{\frac{k \varepsilon_e}{\varepsilon k_e}}$			$Sc_t = \frac{C_\mu f_\mu}{C_\lambda f_\lambda} \sqrt{\frac{k \varepsilon_f}{\varepsilon k_f}}$			
<ul style="list-style-type: none"> • Energy Variance $k_e = \widetilde{e''e''}$ • Dissipation Rate ε_e 			<ul style="list-style-type: none"> • Mixture Fraction Variance $k_f = \widetilde{f''f''}$ • Dissipation Rate ε_f 			
C _{d1} = 2.0	C _{d2} = 0.0	C _{d3} = 0.72	C _{d4} = 2.2	C _{d5} = 0.8	σ _{k,e} = 1.0	σ _{ε,e} = 1.0

Hot (800K) Supersonic Jet - SFM vs LES



SCHOLAR COMBUSTION EXPERIMENT



Turbulent Prandtl Number

Turbulent Schmidt Number

Compressible Subgrid Kinetic Energy Closure

- Transport of the subgrid kinetic energy

$$\frac{\partial}{\partial t} \bar{\rho} k^{sgs} + \frac{\partial}{\partial x_i} (\bar{\rho} \tilde{u}_i k^{sgs}) = \mathcal{T}_{k^{sgs}} + pd_{k^{sgs}} + P_{k^{sgs}} - D_{k^{sgs}}$$

- Production

$$P_{k^{sgs}} = -\tau_{ij}^{sgs} \frac{\partial \tilde{u}_j}{\partial x_i} \quad \tau_{ij}^{sgs} = -2\bar{\rho} \nu_t \left(\tilde{S}_{ij} - \frac{1}{3} \tilde{S}_{kk} \delta_{ij} \right) + \frac{2}{3} \bar{\rho} k^{sgs} \delta_{ij}$$

- Dissipation

- Diffusion/Transport

$$D_{k^{sgs}} = \left(\overline{\tau_{ij} \frac{\partial u_i}{\partial x_j}} - \overline{\tau_{ij}} \frac{\partial \tilde{u}_i}{\partial x_j} \right)$$

- Pressure-Dilatation Correlation

$$\mathcal{T}_{k^{sgs}} = -\frac{\partial}{\partial x_i} \left((\bar{\rho} \widetilde{K} u_i - \bar{\rho} \widetilde{K} \tilde{u}_i - \tilde{u}_j \tau_{ij}^{sgs}) + (\overline{u_i P} - \tilde{u}_i \bar{P}) - (\overline{u_j \tau_{ij}} - \tilde{u}_j \overline{\tau_{ij}}) \right)$$

$$pd_{k^{sgs}} = \overline{P \frac{\partial u_i}{\partial x_i}} - \bar{P} \frac{\partial \tilde{u}_i}{\partial x_i}$$

Genin and Menon (AIAA-2009, Comp. Fl., 2010; J. Turb., 2010)

Day 2, Lecture 8, Suresh Menon, Georgia Tech

Closure for Compressible Flows

- Diffusion of k^{sgs} due to pressure fluctuations transfers acoustic energy from shock front corrugation to subgrid kinetic energy

$$\overline{u_i P} - \tilde{u}_i \overline{P} = \bar{\rho} \tilde{R} (\widetilde{u_i T} - \tilde{u}_i \tilde{T}) = -\frac{\bar{\rho} \nu_t \tilde{R}}{Pr_t} \frac{\partial \tilde{T}}{\partial x_i}$$

- Subgrid pressure – dilatation correlation

$$pd_{k^{sgs}} = \alpha_{pd} M_t^{sgs 2} \left(\frac{\bar{\rho} \tilde{S} k^{sgs}}{D_{k^{sgs}}} \right)^2 (P_{k^{sgs}} - D_{k^{sgs}})$$

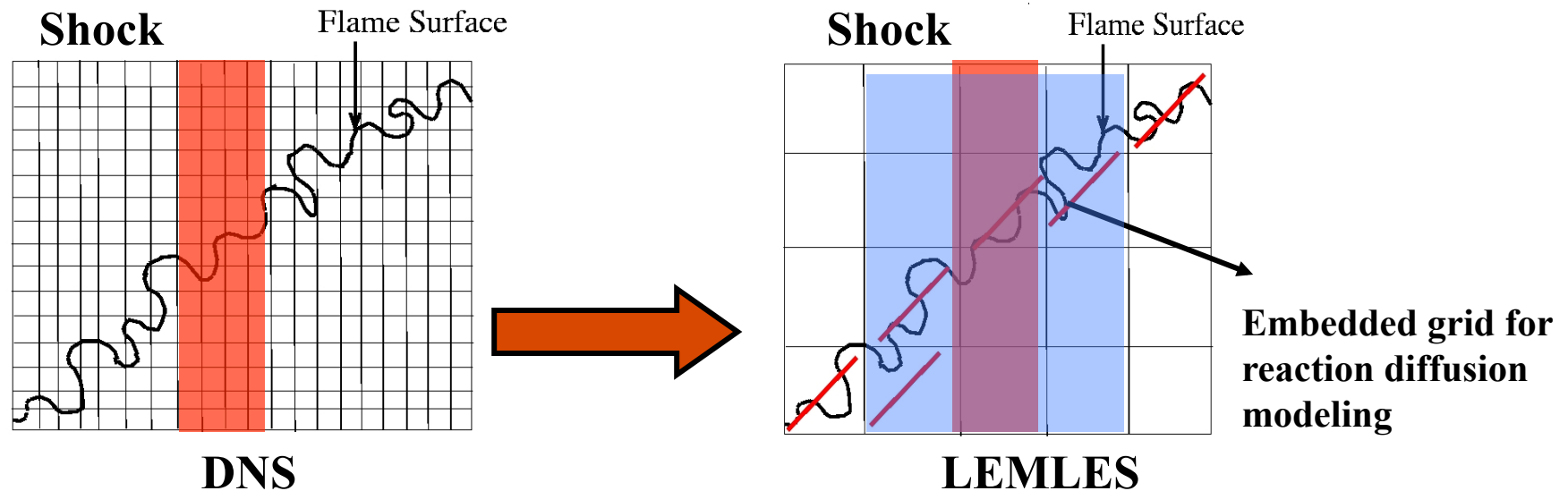
- Energy Equation closure (turbulent Prandtl number)

$$H_i^{sgs} + \sigma_i^{sgs} = -(\bar{\rho} \nu_t + \mu) \frac{\partial k^{sgs}}{\partial x_i} - \frac{\bar{\rho} \nu_t}{Pr_t} \frac{\partial \tilde{T}}{\partial x_i} + \tilde{u}_j \tau_{ij}^{sgs}$$

Localized Dynamic Evaluation

- Extension of LDKM (Kim and Menon, 1995, 1991) for low-speed flows to compressible flows
 - Genin and Menon, *Comp. Fl.* (2010), *JoT* (2010)
- SGS closure model constants obtained from shock-turbulence DNS/LES comparison (*Comp. Fl.*, 2010)
- Dynamic closure using scale similarity at the test filter level
 - Numerically robust and stable in complex flows
- Localized dynamic evaluation of Pr_t can be used to close
 - Subgrid energy diffusion in the energy equation
 - Diffusion of k^{sgs} due to pressure fluctuations
- Localized dynamic evaluation of Sc_t (if not using LEM)
- NO model parameters that are adjusted to match test case

LEMLES: Grid-Within-Grid Approach



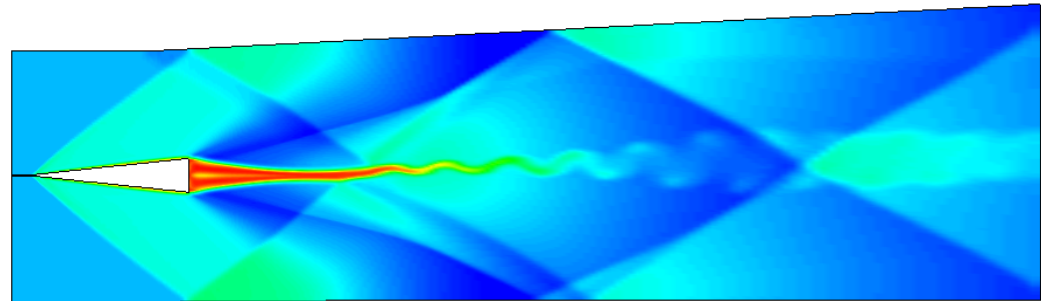
- **Multi-scale (space and time) approach (LEMLES)**
- **Application to subsonic turbulent reacting flows since 2000**
 - **No ad hoc model constant adjustments**
- **Extension to shock-turbulence-flame interaction problems**
 - **LEM updated to allow for subgrid pressure variation**
 - **subgrid compression and expansion**
 - **Explicit presence of shock in subgrid not yet included**

Hybrid Numerical Algorithm

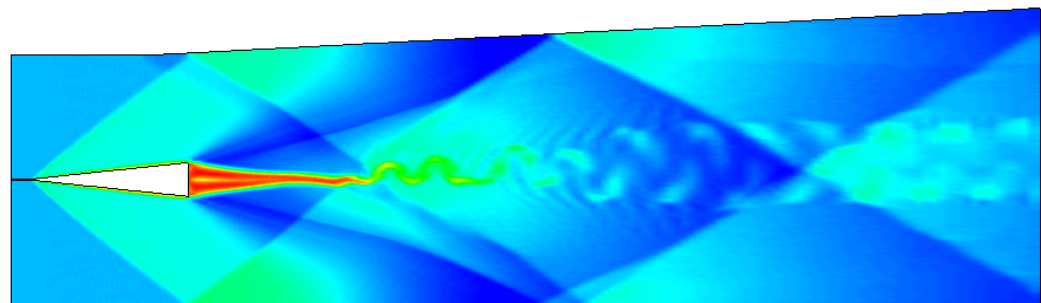
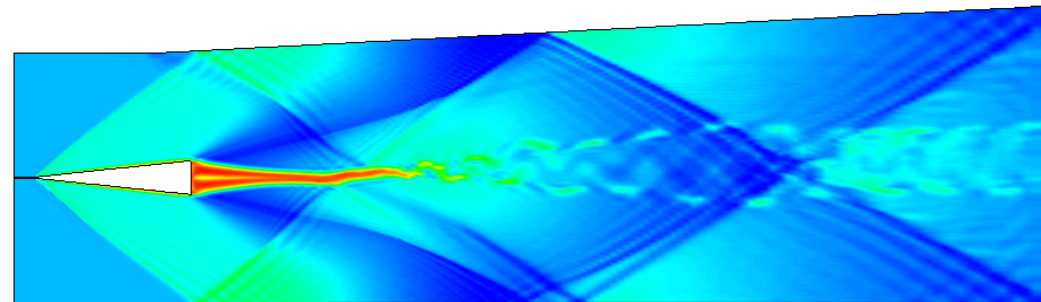
- Locally adaptive hybrid strategy switches from shock capturing solver to a smooth-flow ($O(4)$) solver locally and dynamically
- Piecewise Parabolized Method (PPM – FLASH3D)
 - Extended to viscous flows, multi-domain, stretched grids
- MUSCL reconstruction with Hybrid HLL Riemann Solver
 - Non-contact preserving in shock transverse directions (HLLE, Einfeldt, 1988, 1991)
 - Contact preserving Riemann solver (HLLC, Toro, 1997)
- The current hybrid solver is identified as 4th/HLLC/E
 - Smoothness local sensors to switch between $O(4)$ & HLL
 - Local shock detection to switch from HLLC to HLLE

Numerical scheme and accuracy

- Illustration for Scramjet flowfield: Supersonic airflow ($M=2$) over a 6 degrees wedge – vortex street and turbulence



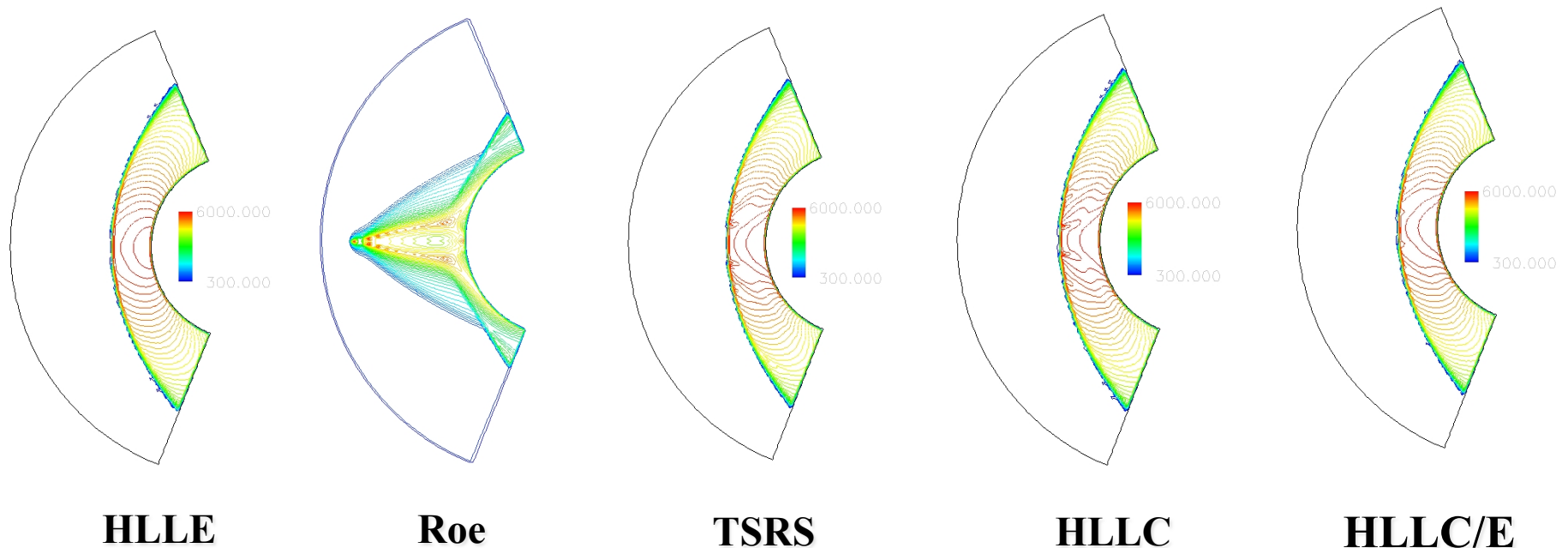
- (1) Pure upwind is dissipative
- (2) Central with artificial dissipation is dispersive
- (3) Hybrid method to switch between numerical schemes



Riemann Solvers Instabilities and Remedy

- Odd-Even decoupling and Carbuncle phenomenon arise in numerical resolution of shock waves
 - neighboring mesh points along a shock front decouple
 - strongly deform shock fronts and creates parasitic oscillations in the post-shock region
- Design of a hybrid Riemann solver – Extension of Quirk's cure to Riemann solvers: use of a non-contact preserving Riemann solver in the directions transverse to the shock normal
- Flattening (reduce reconstruction order close to strong shocks) to prevent post-shock oscillations

Carbuncle Test Case

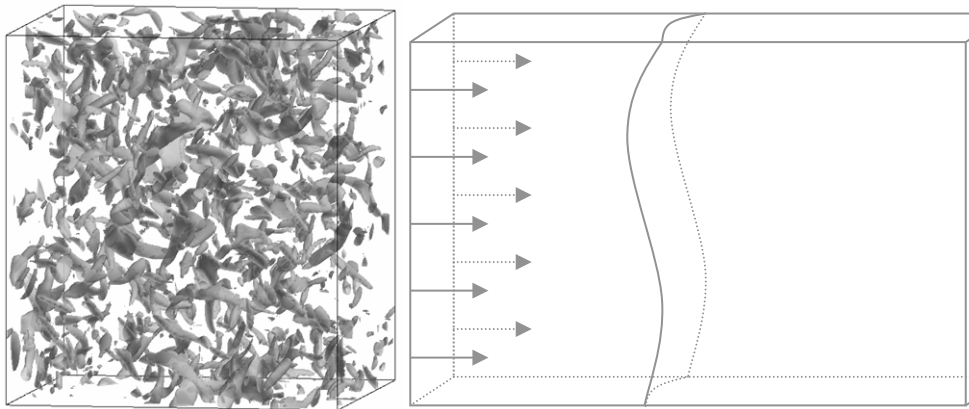


- M=10 air flow onto a cylindrical body
- Appearance of singular points for contact-preserving solvers
 - not seen for HLLE
- Even more reduced effect with HLLC/E

Normal Shock-Turbulence Interaction

- DNS (231x81x81) and LES (106x32x32)
- Isotropic turbulence (243x81x81) superposed on supersonic inflow:

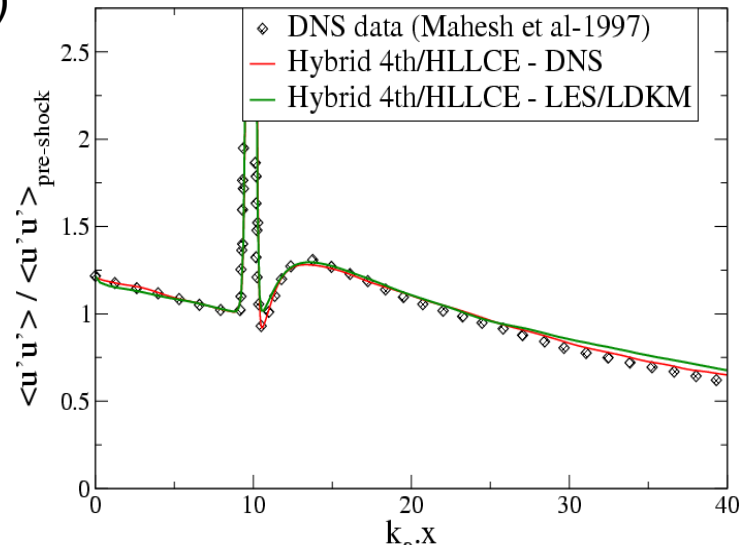
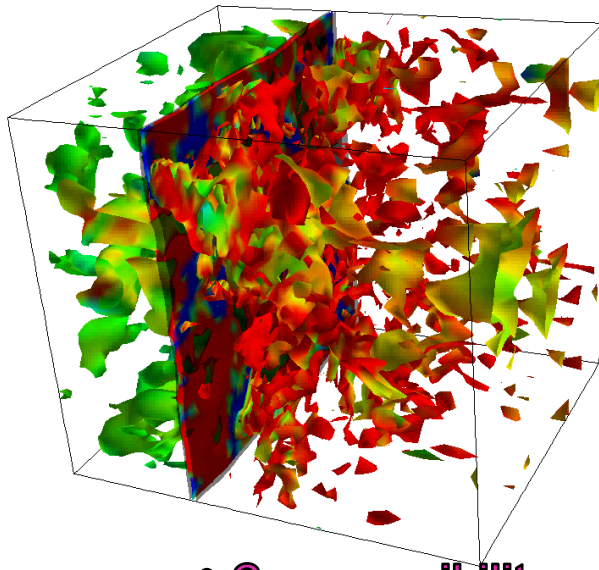
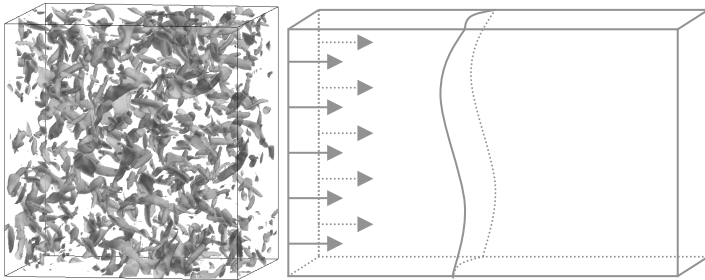
M_{inflow}	1.29	2	3	
Re_{λ}		19.1	19.0	19.7
M_t		0.140	0.108	0.110



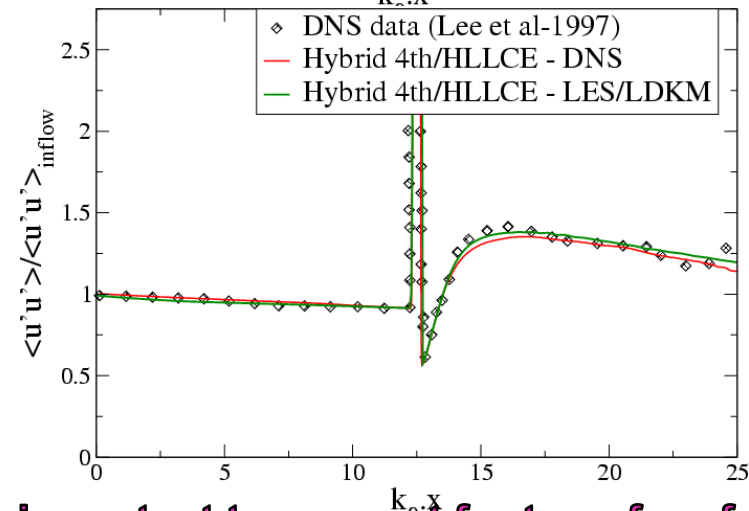
- Variations of local Mach number → Shock corrugation
- Post-shock pressure fluctuations, acoustic wave
- Exchange between acoustic energy & turbulent kinetic energy

Shock-Turbulence Interaction

- DNS (231x81x81) LES (106x32x32)
- Dynamic LES closure captures turbulence across shock



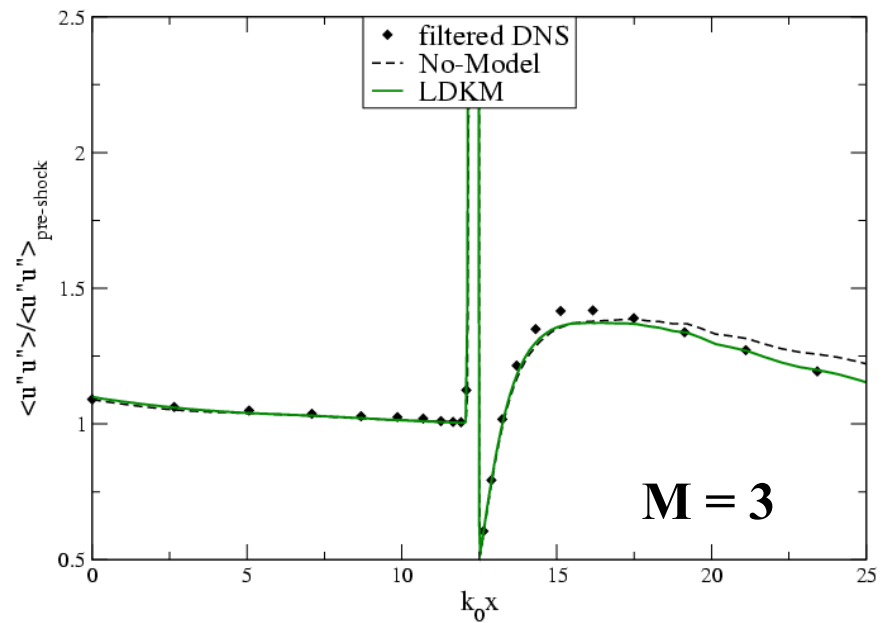
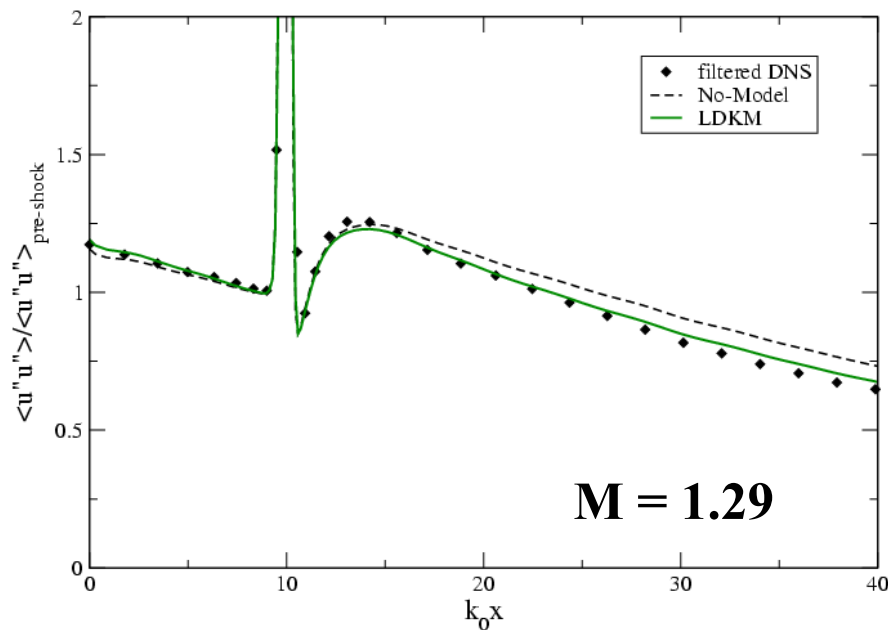
M = 1.29



M = 3.0

- Compressibility correction is important to account for transfer of acoustic energy from shock corrugations to sub-grid kinetic energy

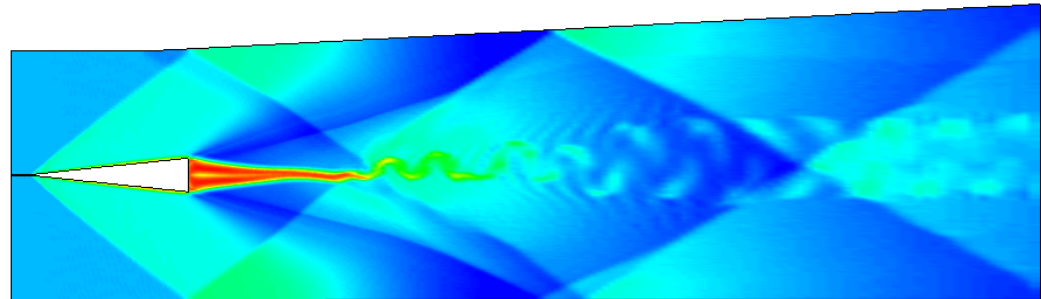
Normal Shock / Turbulence Interaction



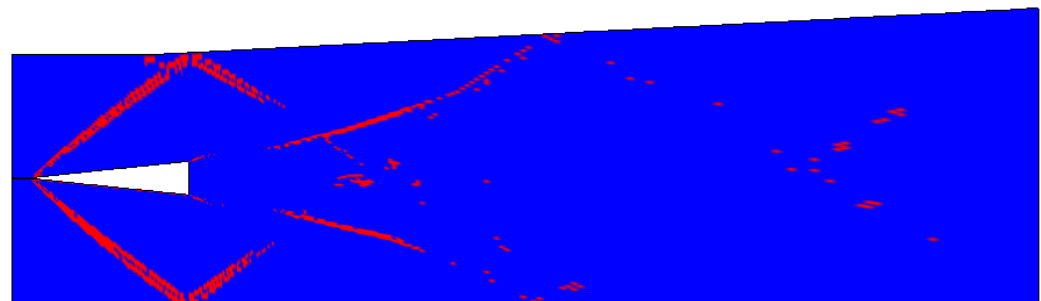
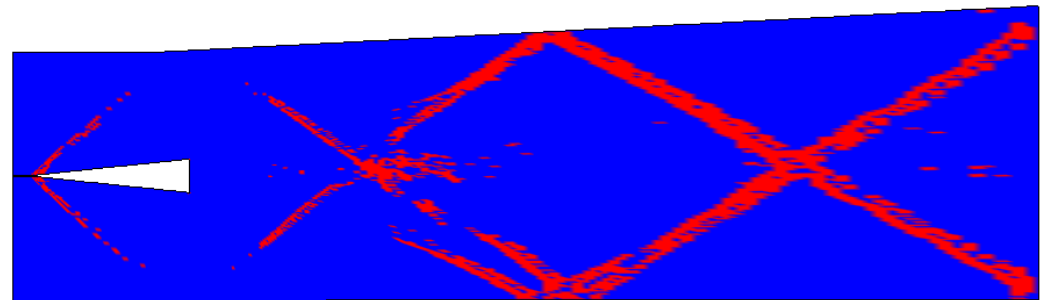
- LES captures most of the DNS features
- Dynamic model shows stable predictions for all simulated M
- Compressibility corrections appears to work well

Numerical scheme and accuracy

- Proper capture of the flow discontinuities with upwind scheme and resolution of the instability and turbulence

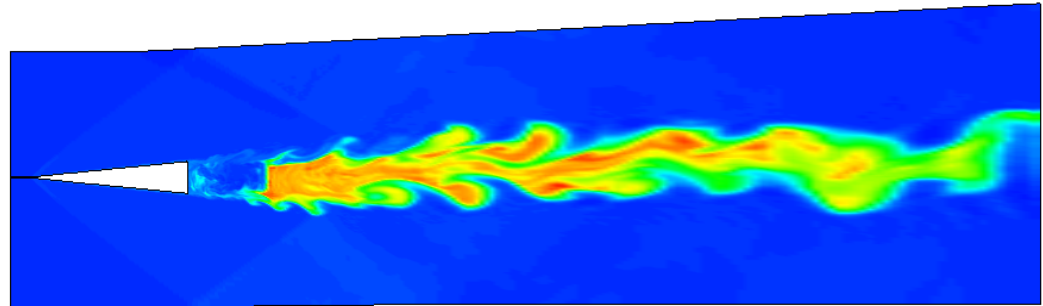


- (1) Temperature field
- (2) Use of upwinding in the I-direction
- (3) Use of upwinding in the J-direction

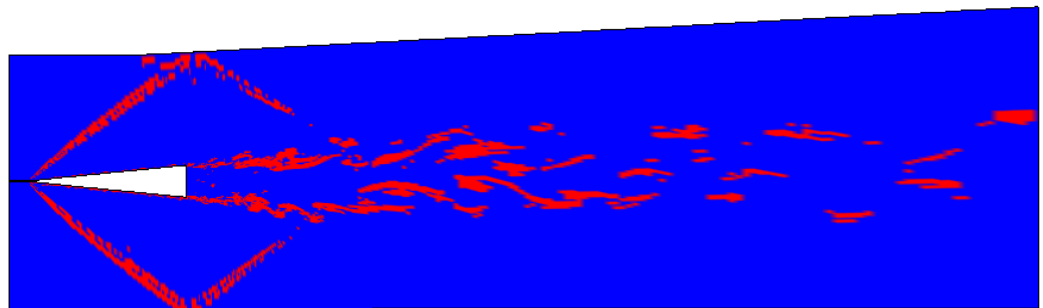
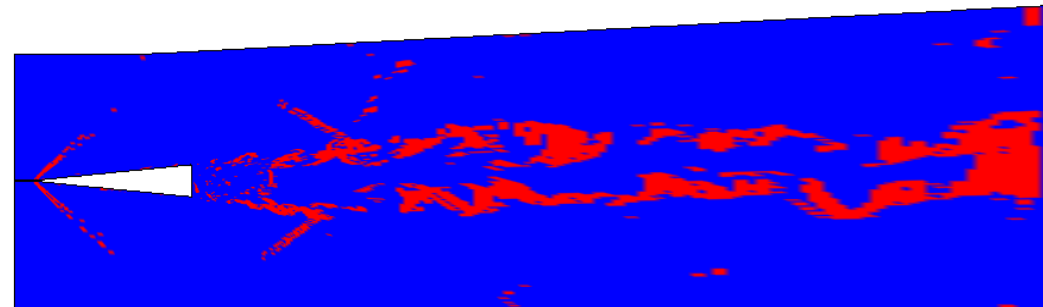


Numerical scheme and accuracy

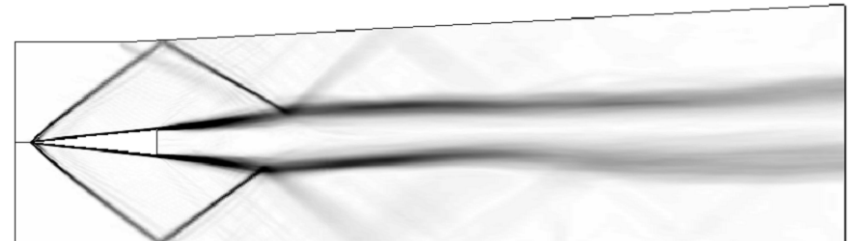
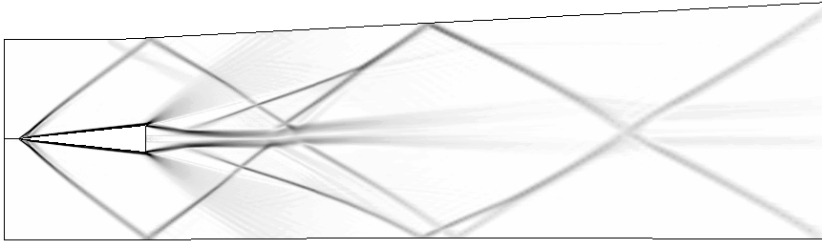
- With reaction, more sharp fronts
- Sonic injection of H_2 at the base of the wedge



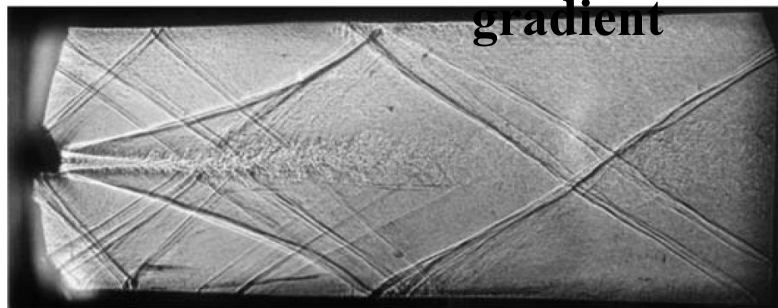
- (1) Temperature field
- (2) Use of upwinding in the I-direction
- (3) Use of upwinding in the J-direction



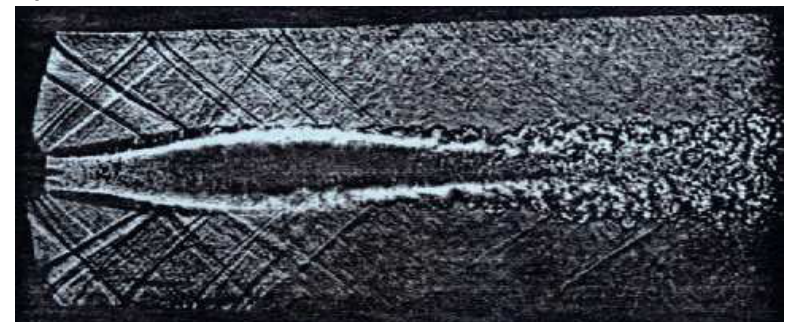
Non-Reacting and Reacting DLR Test Case



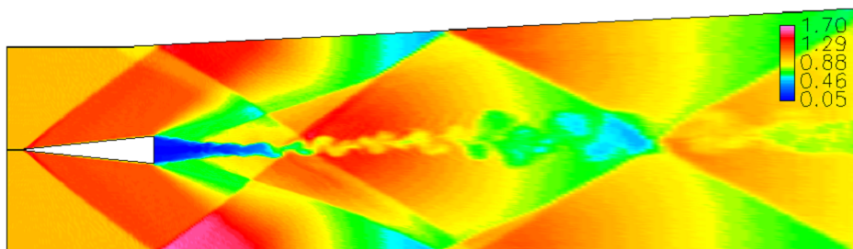
Time-averaged density



gradient

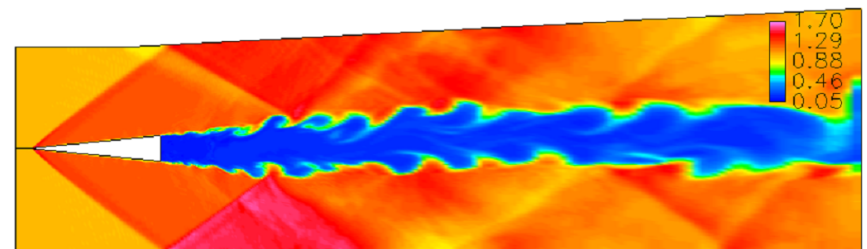


Experimental Schlieren



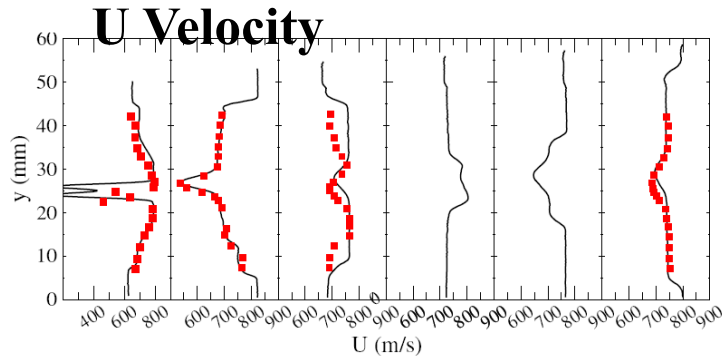
COLD

Instantaneous density

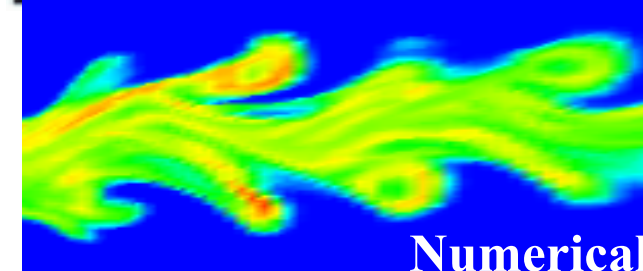
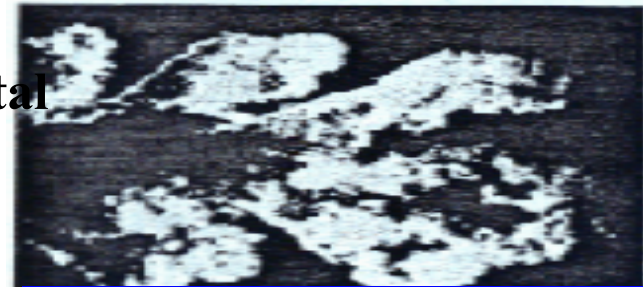


HOT

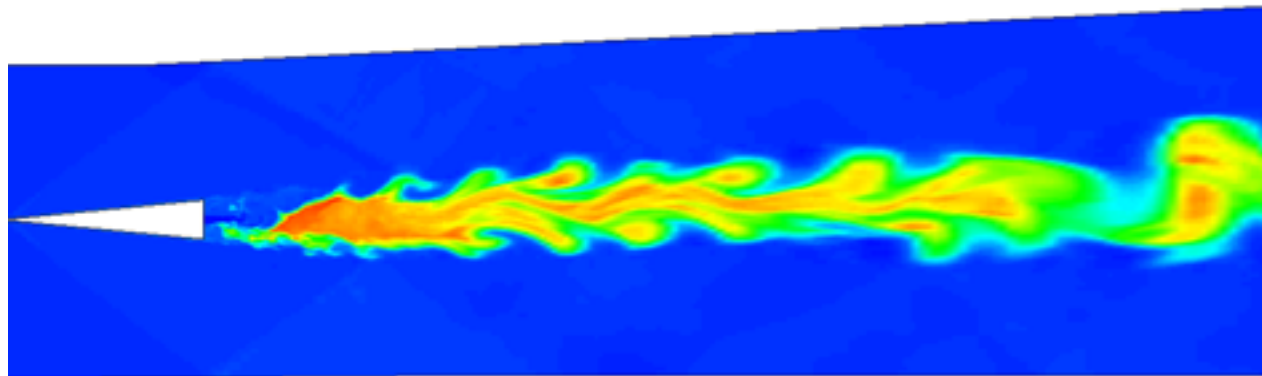
Comparisons with DLR Data



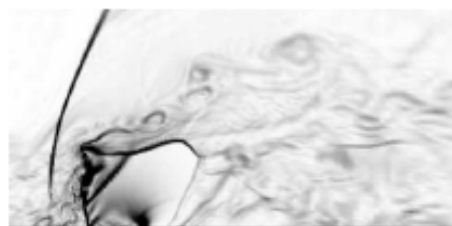
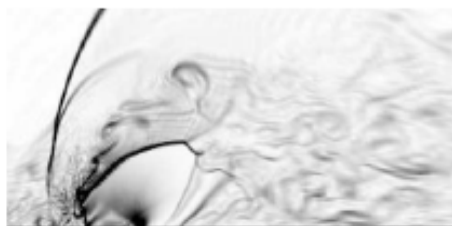
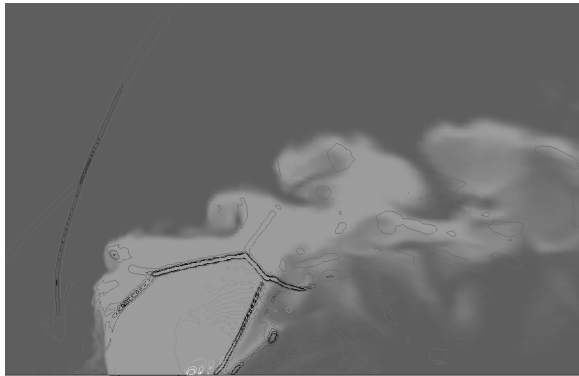
**Experimental
OH-PLIF**



- **Flame anchors by re-circulation of hot products with intermittent reverse flows**
- **Partially premixed ignition**
- **Diffusion flame along the shear layer**



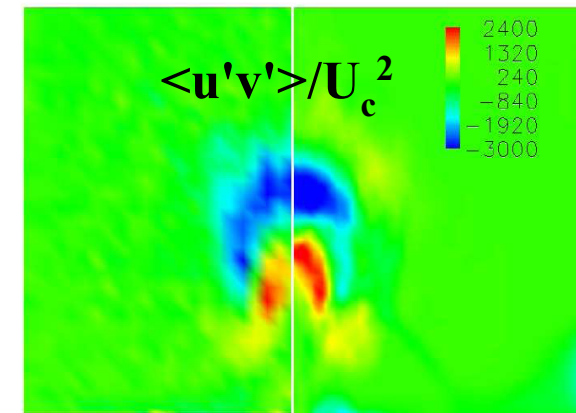
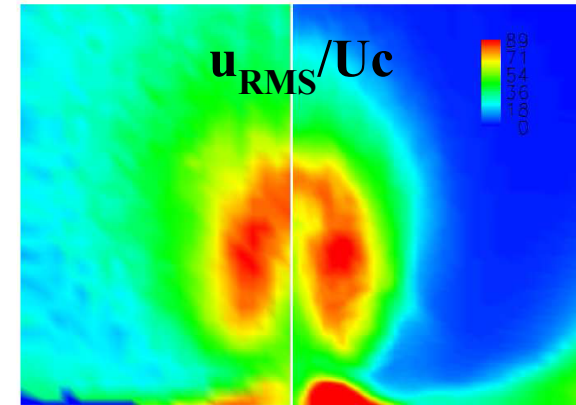
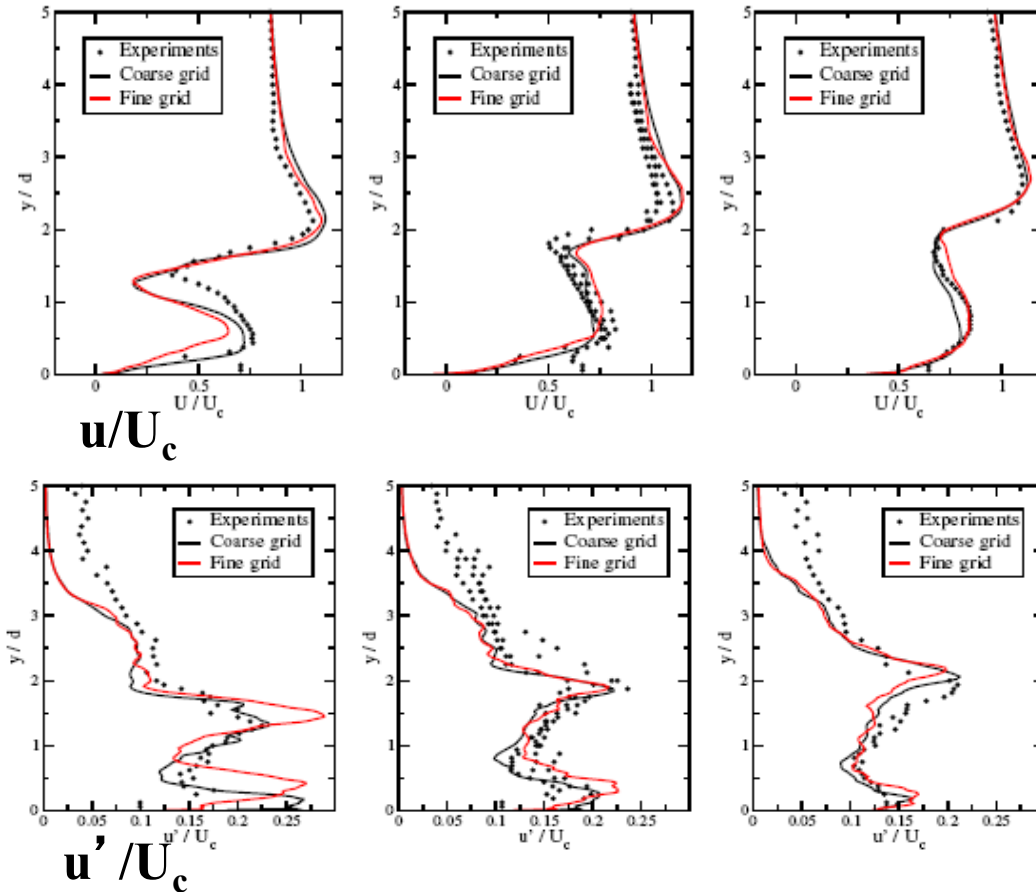
Sonic Jet in M=1.6 Cross-Flow



Current LES

Flow Vis (vanLarberghe, 2000)

Comparison with Experiments

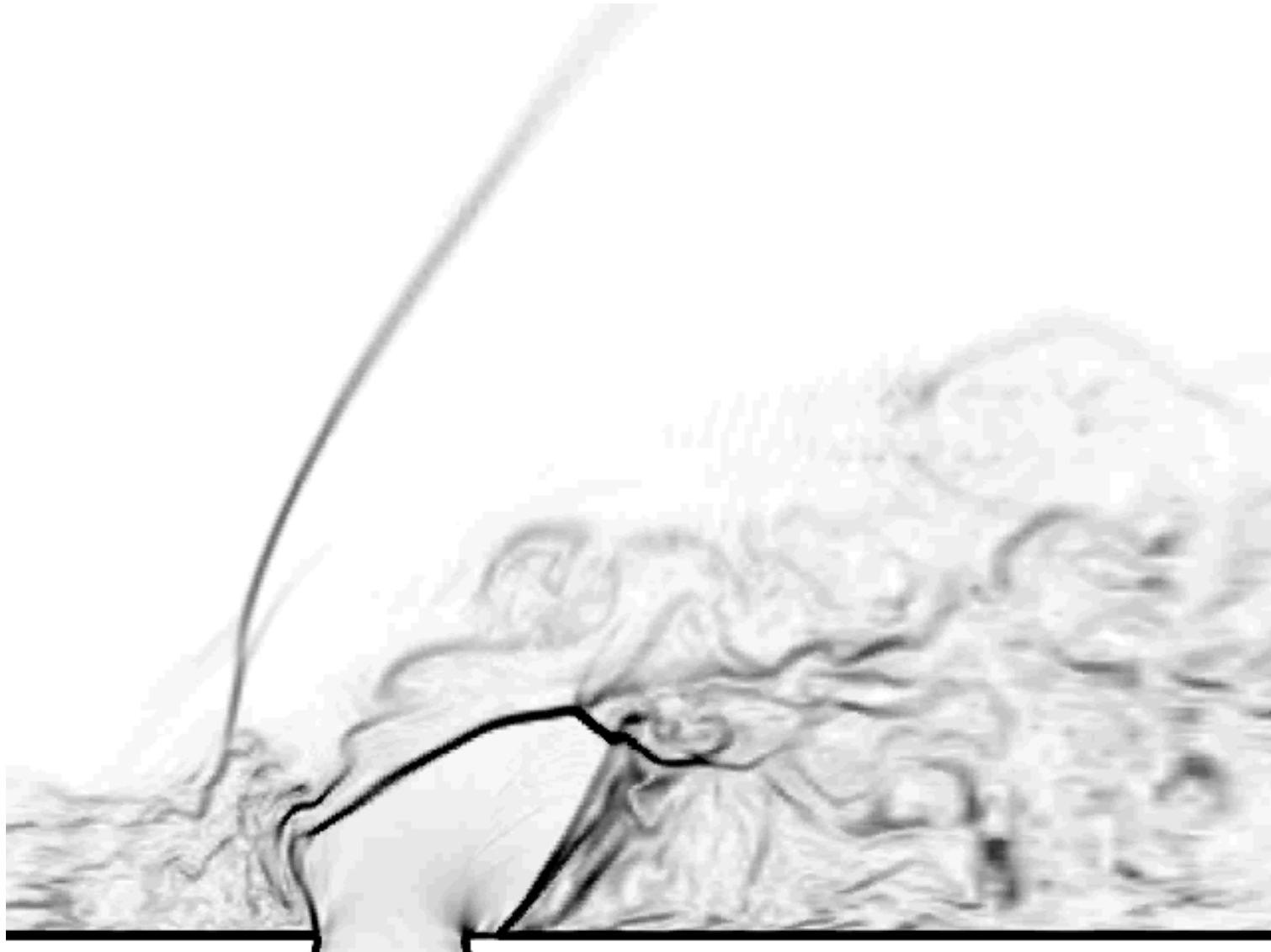


Expt. (x/d=5) LES

Santiago and Dutton (JPP, 1997)

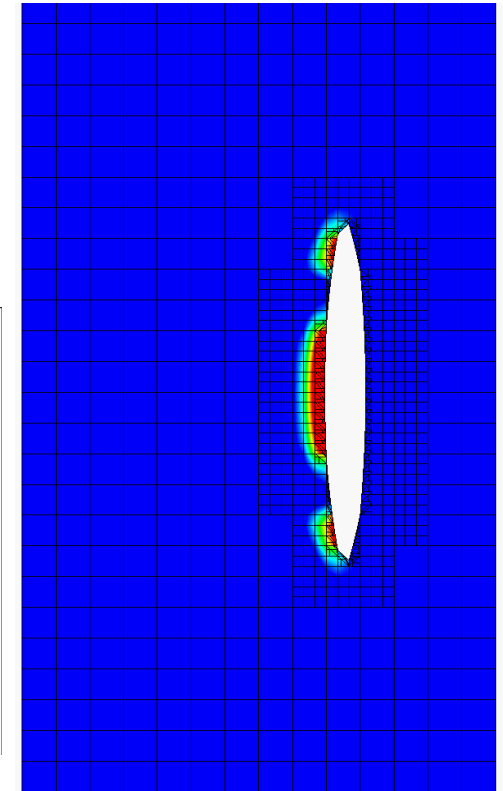
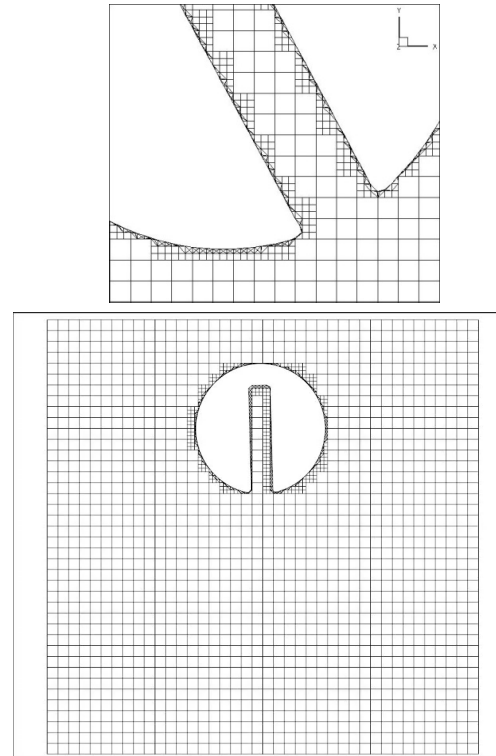
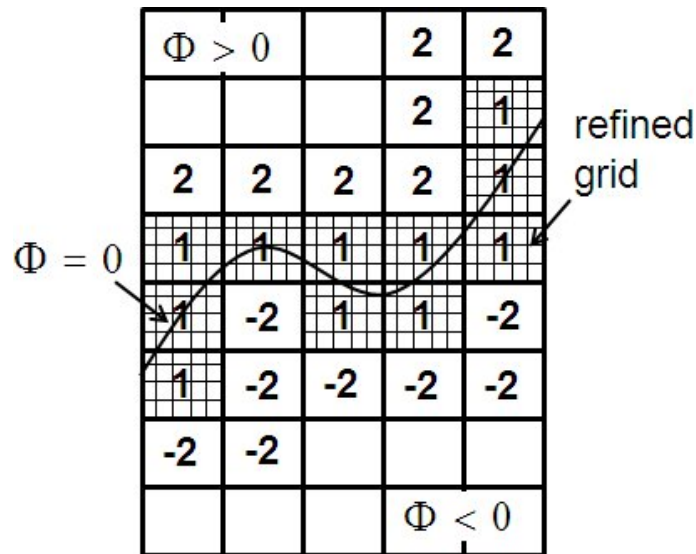
Day 2, Lecture 8, Suresh Menon, Georgia Tech

Supersonic JICF



Adaptive Refined Level Set in LESLIE

- Application to moving shocks, flames and bodies
- ~ AMR for shocks and flames
- Interface tracking and cut-cell for moving bodies



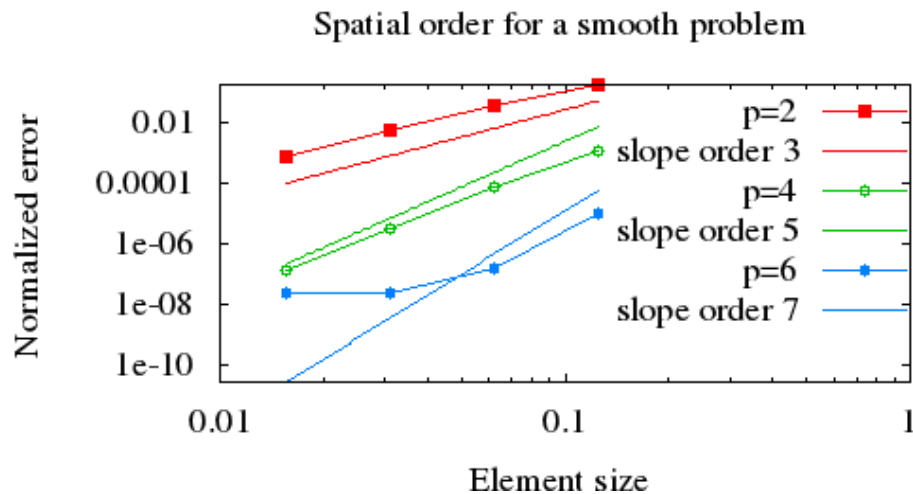
Zaleski Disk

Deforming Ellipsoid in $M = 6$

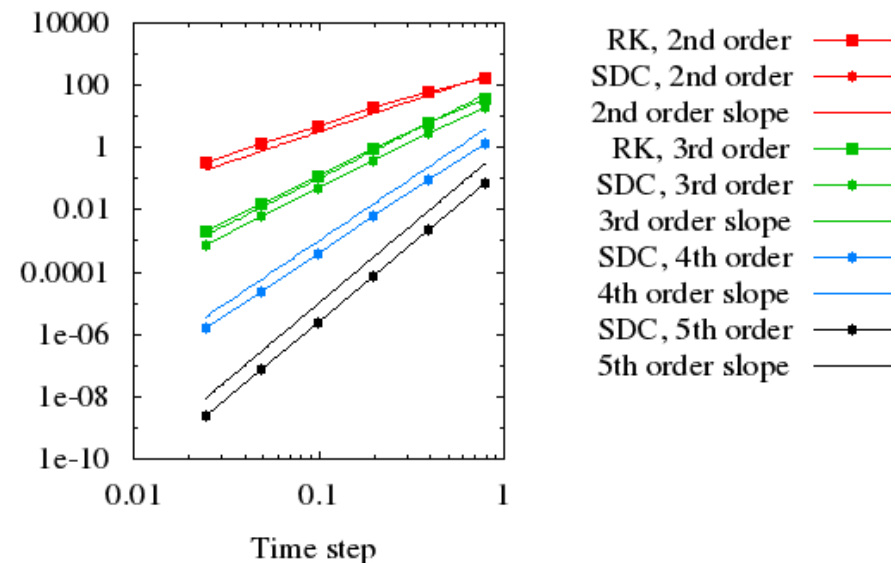
- Osher and Fedkiw, Level Set Methods and Dynamic Implicit Surfaces, 2002
- Choi and Menon, AIAA-2010-414, 2011-417, Georgia Tech

Strategies in DIGGIT

- Compressible 5-equation two-fluid model included
- Time integration with TVD-RK O(3) or SDC (for higher order)
- AMR based on detecting inter-element discontinuity
- For smooth flows: Up to O(7) in space and O(5) in time
- Trouble cell detector to apply moment limiter for shocks

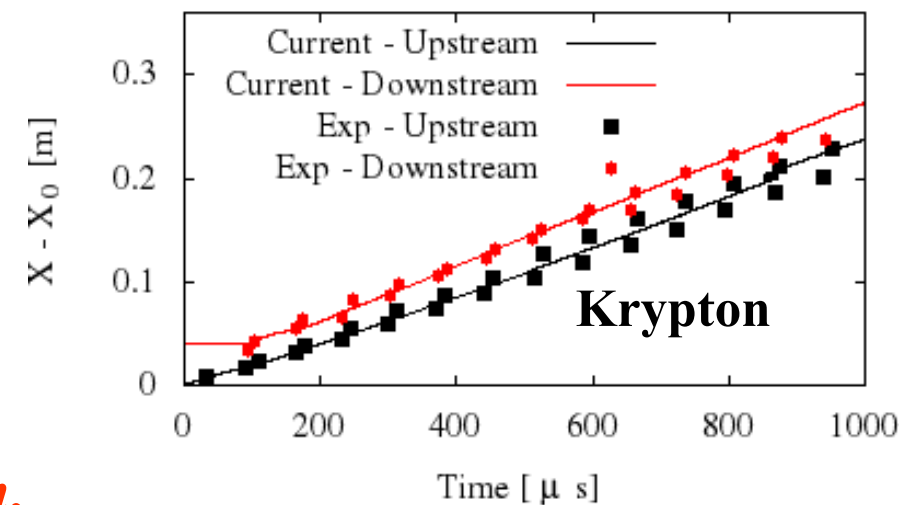
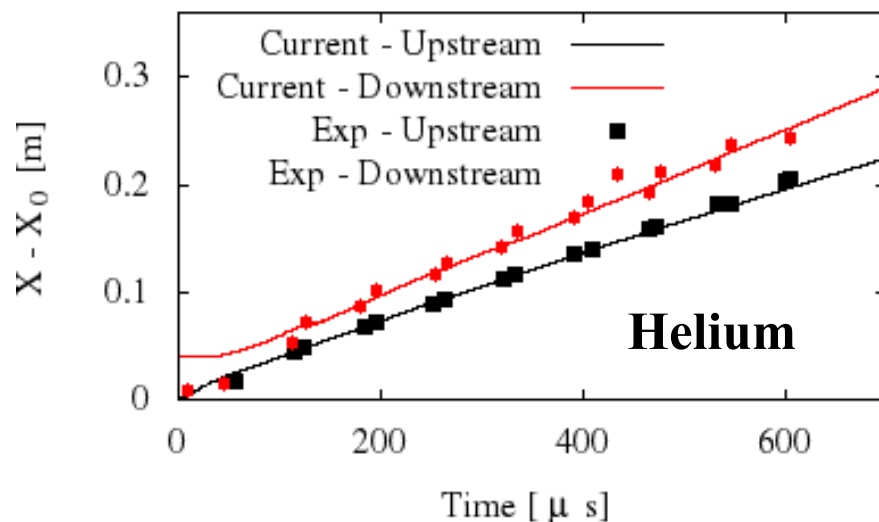
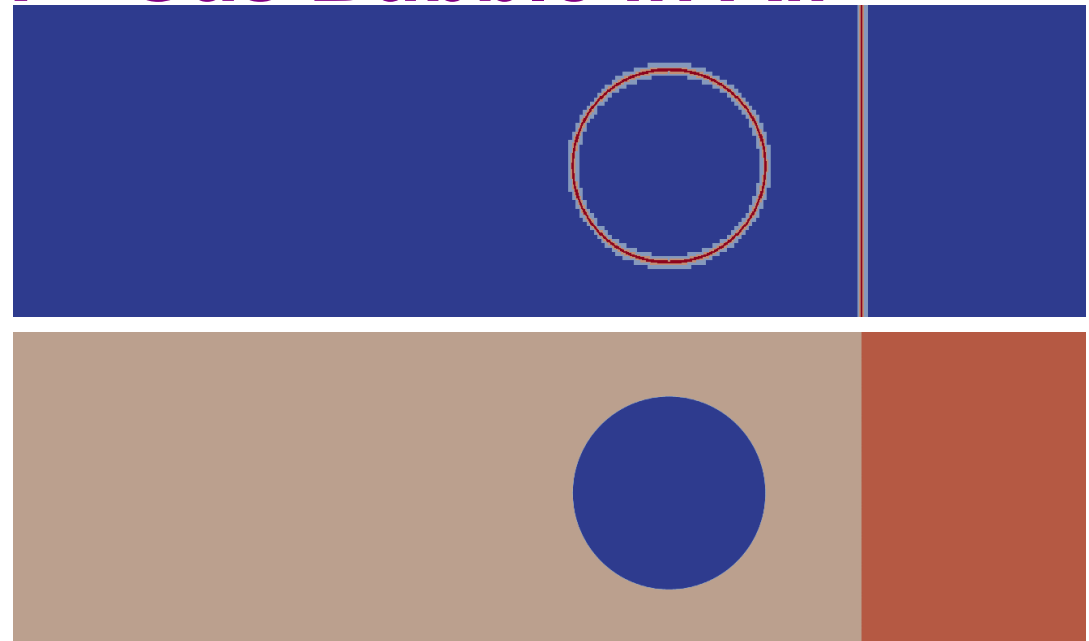


Order in time for non-linear smooth problem



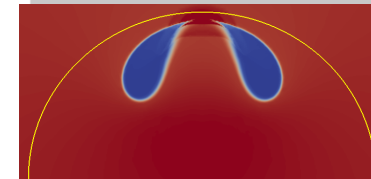
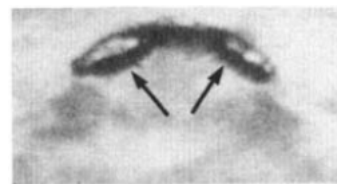
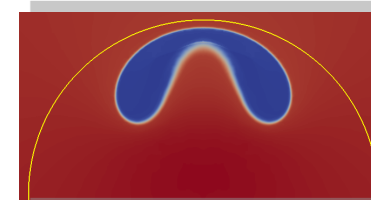
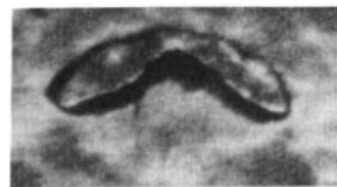
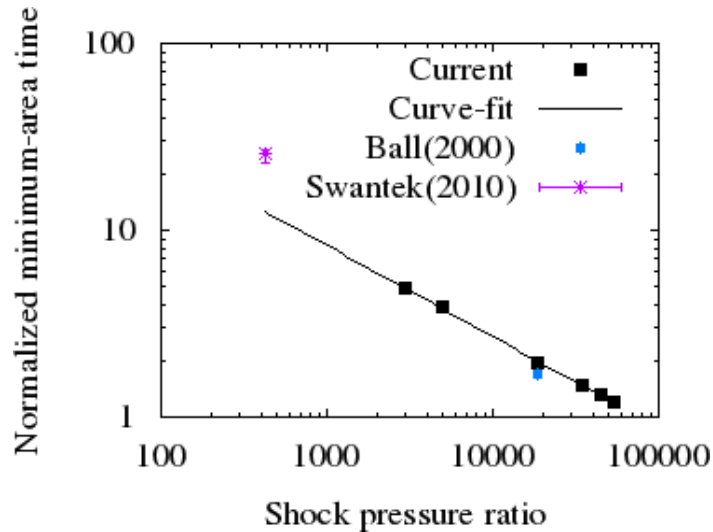
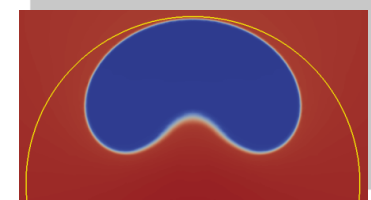
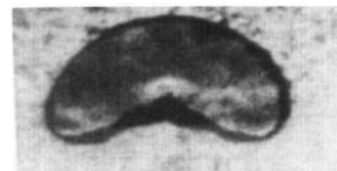
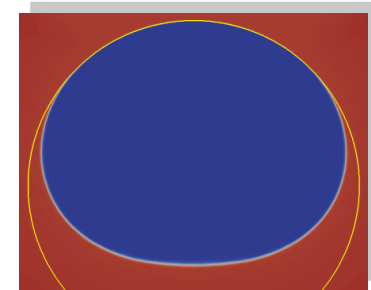
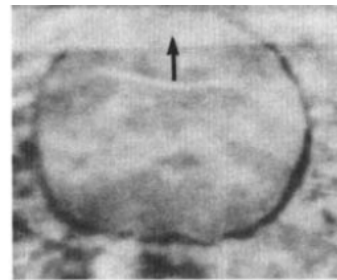
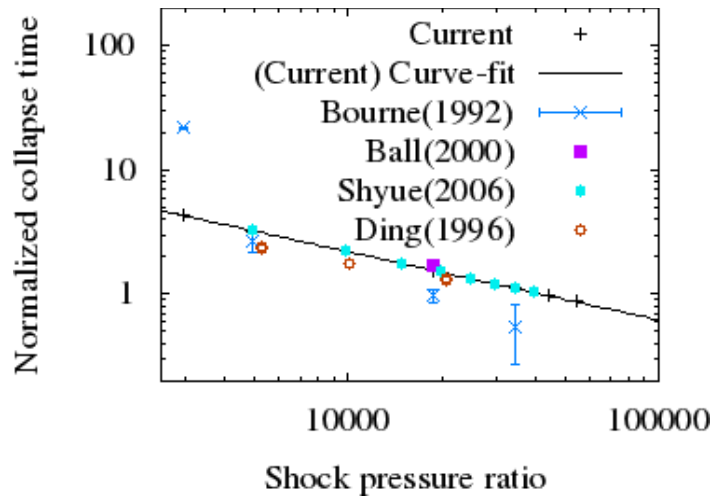
LDG: Shock – Gas Bubble in Air

- M=1.22 He-air 2D cylindrical bubble with 4-level AMR (right)
- M=1.7 Kr-air spherical bubble
- L Hayes and Le Metayer, (Phys. Fl., 2007)

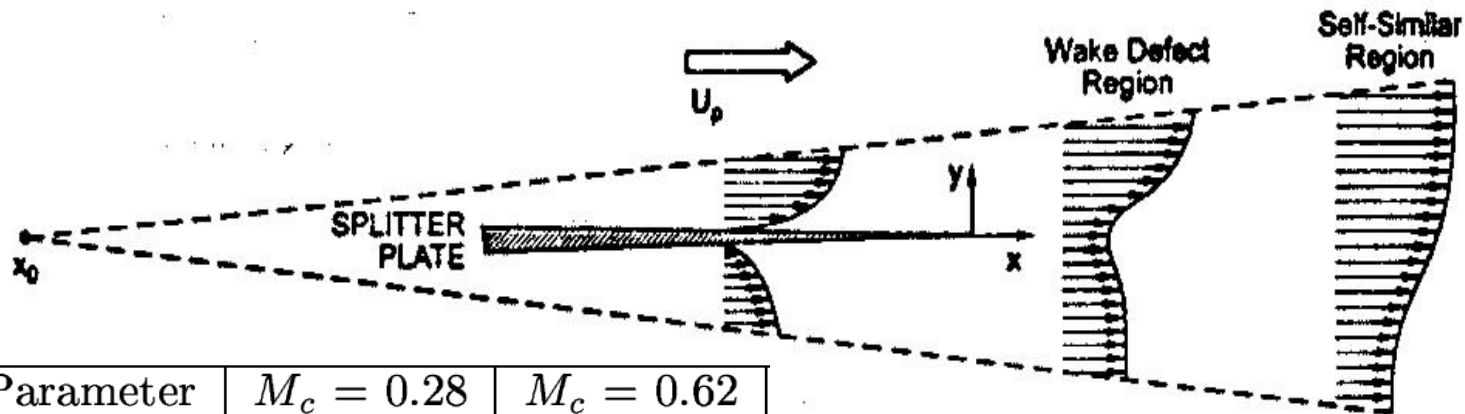


n, ...

LDG: Shock – Air 3D Bubble in Water



Compressible Spatial Shear Layer



Parameter	$M_c = 0.28$	$M_c = 0.62$
M_1	1.64	2.0
M_2	0.91	0.4
U_1 (m/s)	430	480
U_2 (m/s)	275	130
T_1 (K)	172	150
T_2 (K)	223	252
P_{O_1} (Kpa)	302	495
P_{O_2} (Kpa)	115	75
ρ_2/ρ_1	0.77	0.59
U_2/U_1	0.63	0.27
λ_B μm	1.7	0.8
Resolution	$2 - 7 \lambda_B$	$5 - 14 \lambda_B$

• **LEMLES resolution same as experiment deliberately**

* **Passive scalar mixing**

182 x 150 x 5 -

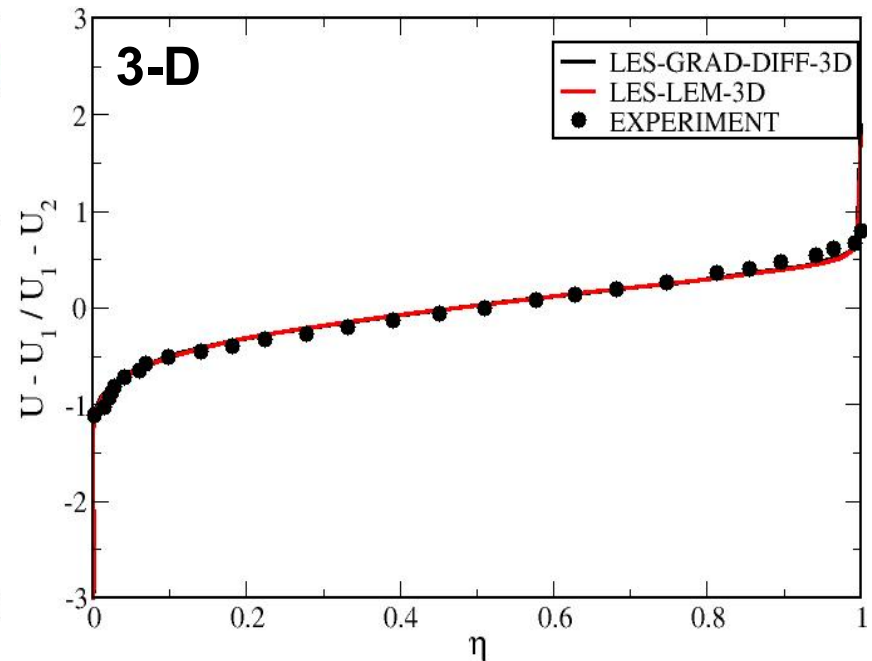
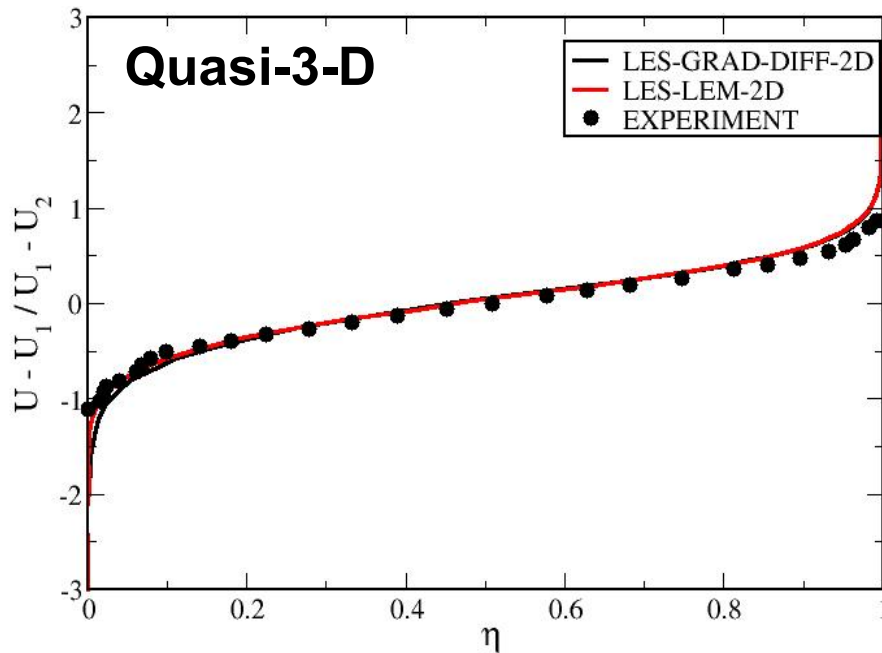
Quasi-3D

182 x 150 x 100 - Full 3D

$y^+ = 15$ @ splitter plate

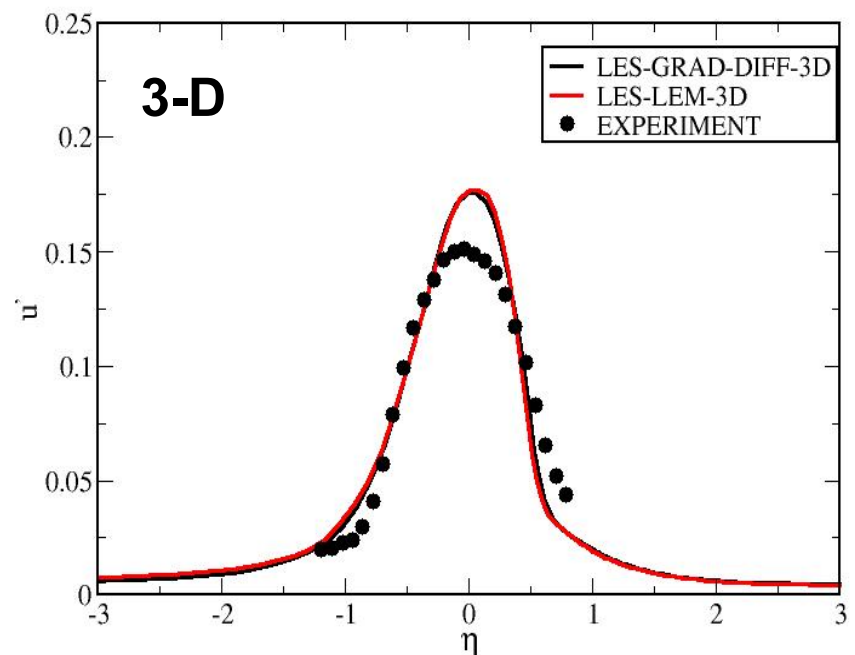
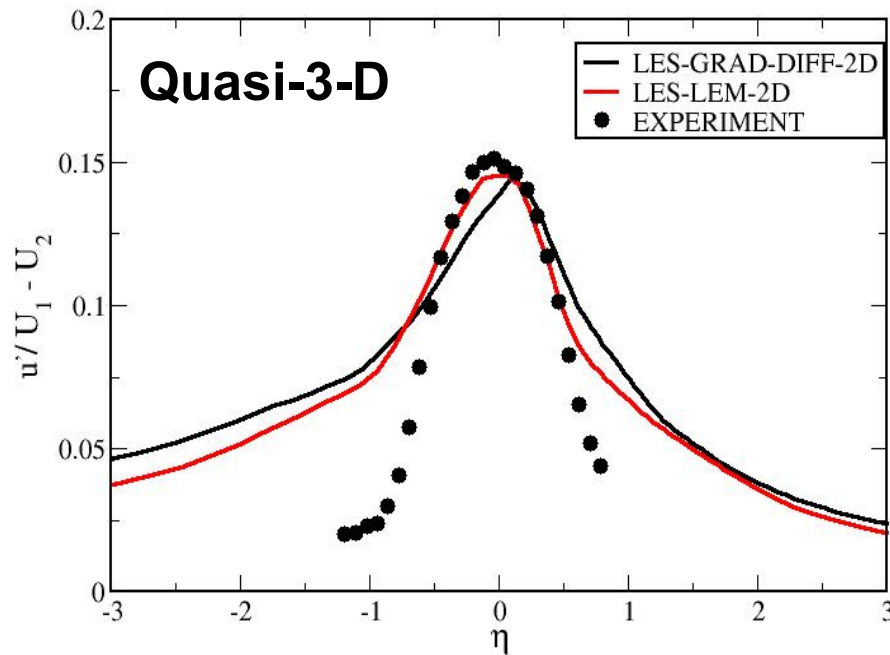
Uniform grid in Z direction

Profiles of Axial Velocity, $Mc = 0.62$



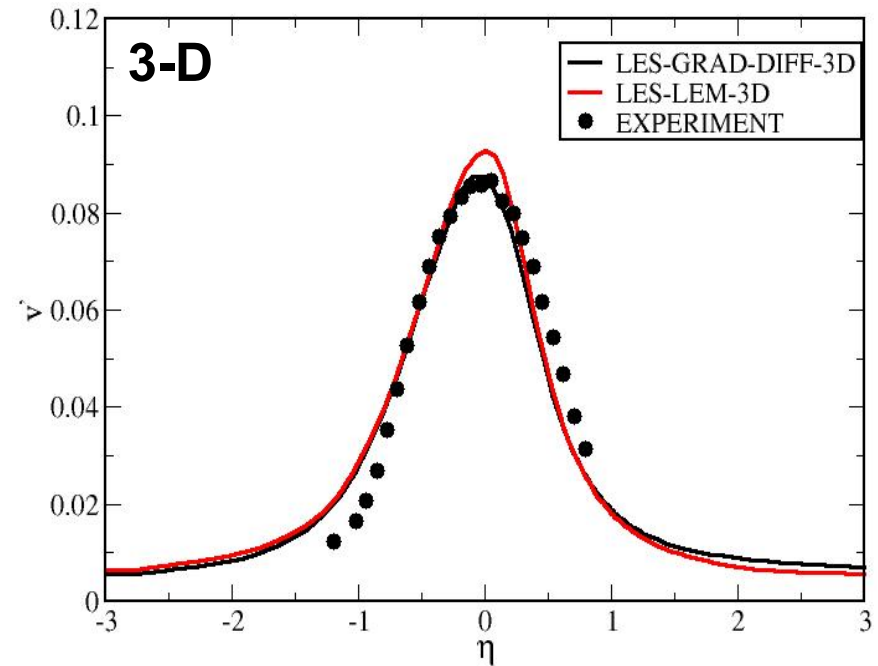
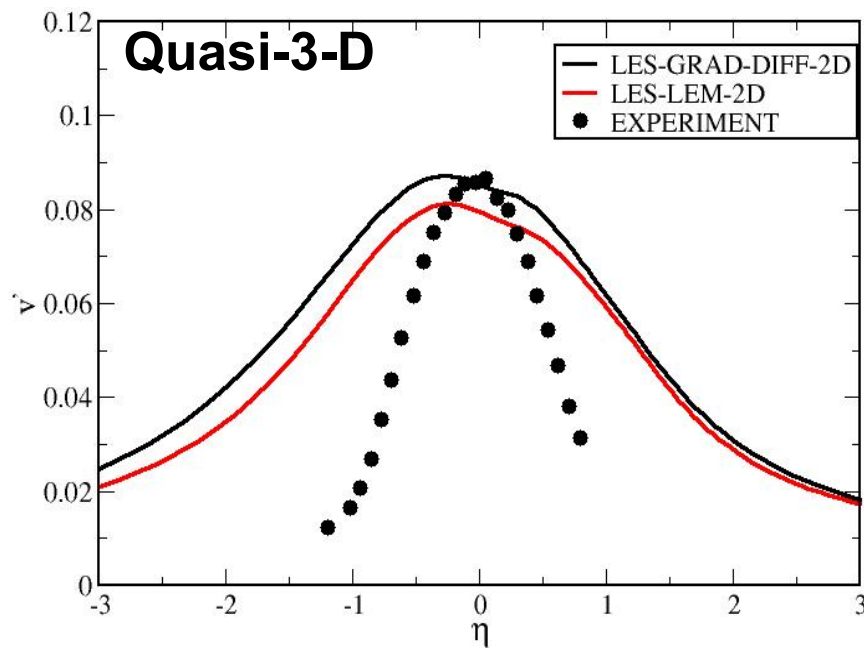
- **GRAD-DIFF and LEMLES employs same k-sgs closure**
- **Both methods agree well with the experiments**

RMS Axial Velocity, $Mc = 0.62$



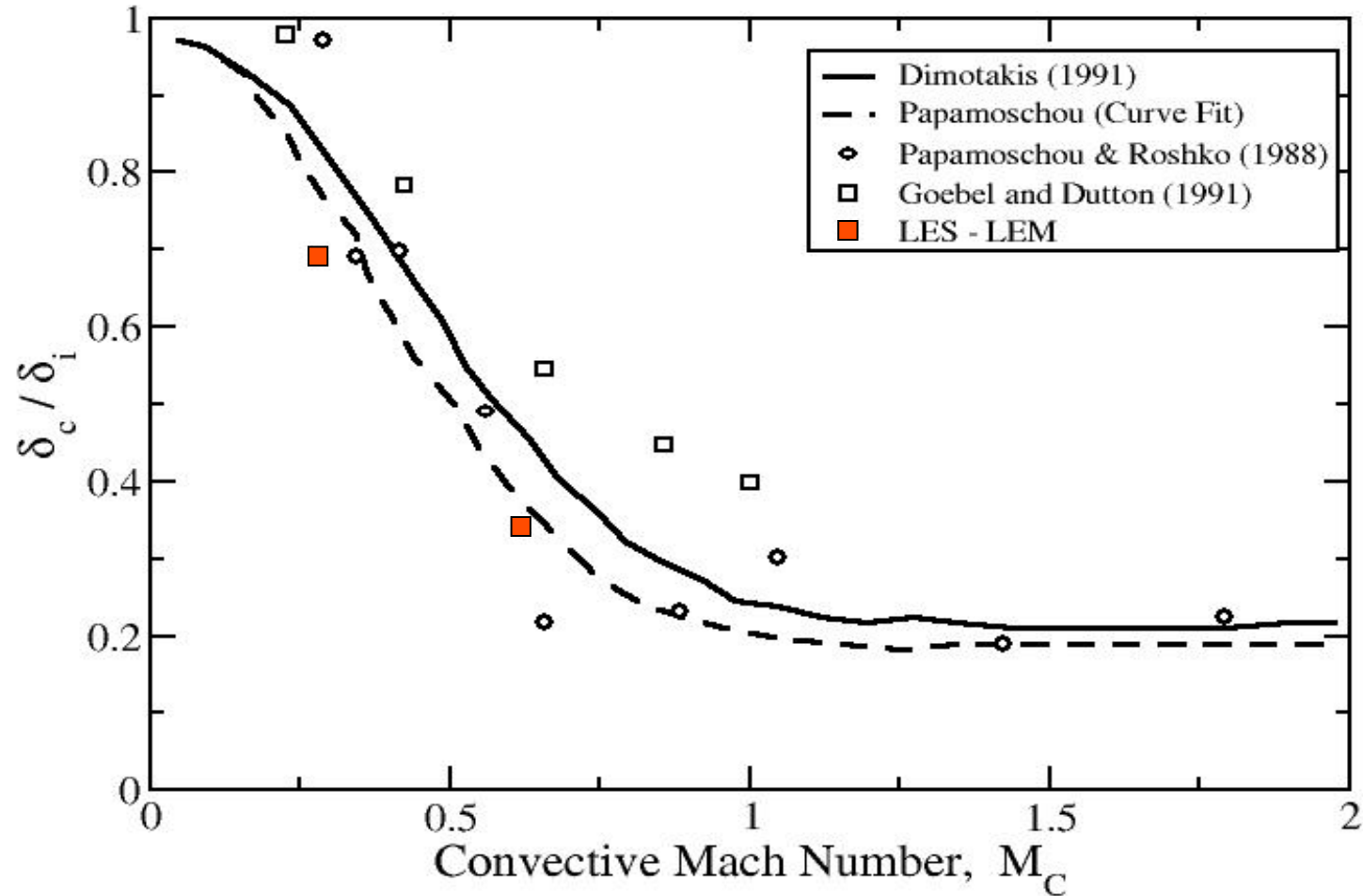
- 3D captures the shear layer spread correctly

RMS Transverse Velocity, $Mc = 0.62$

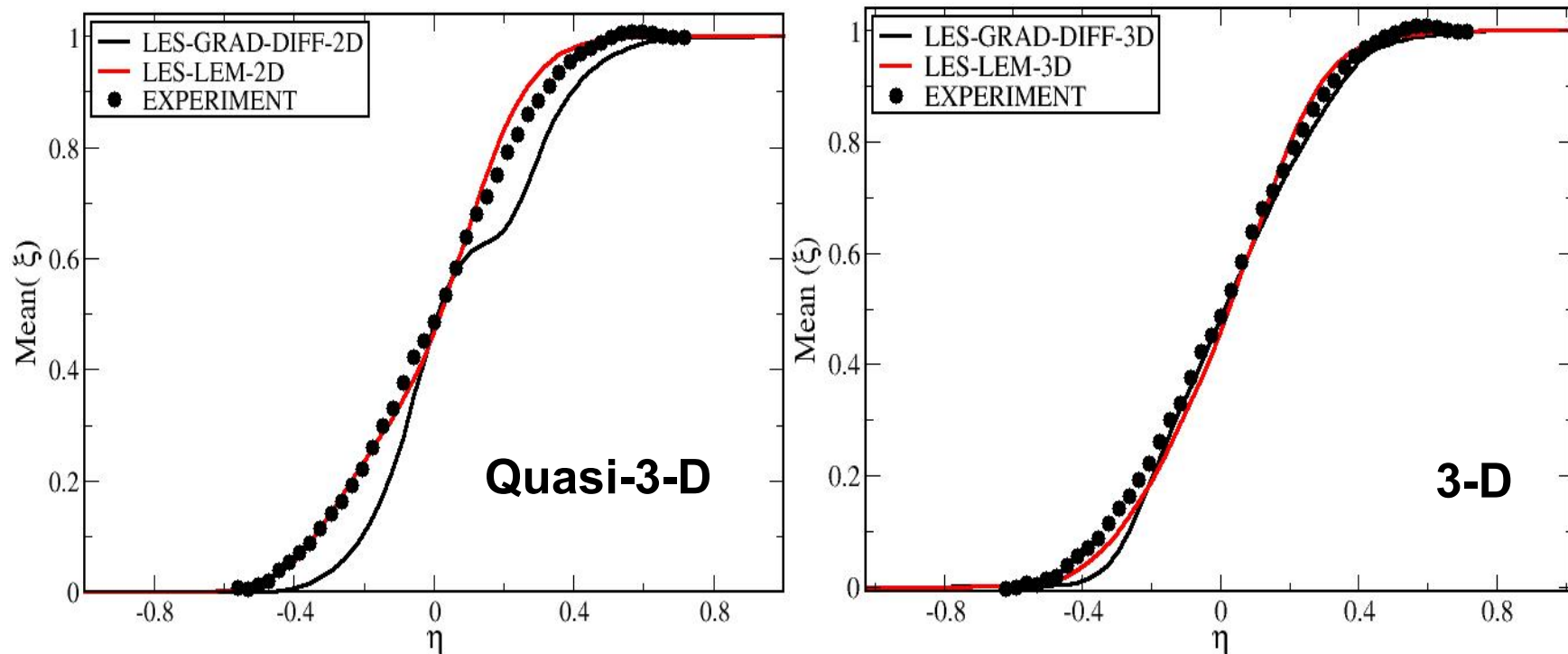


- **GRAD-DIFF and LEMLES over-predicts the peak by $< 6\%$**
- **Good agreement in the shear layer region**

Normalized Growth Rate

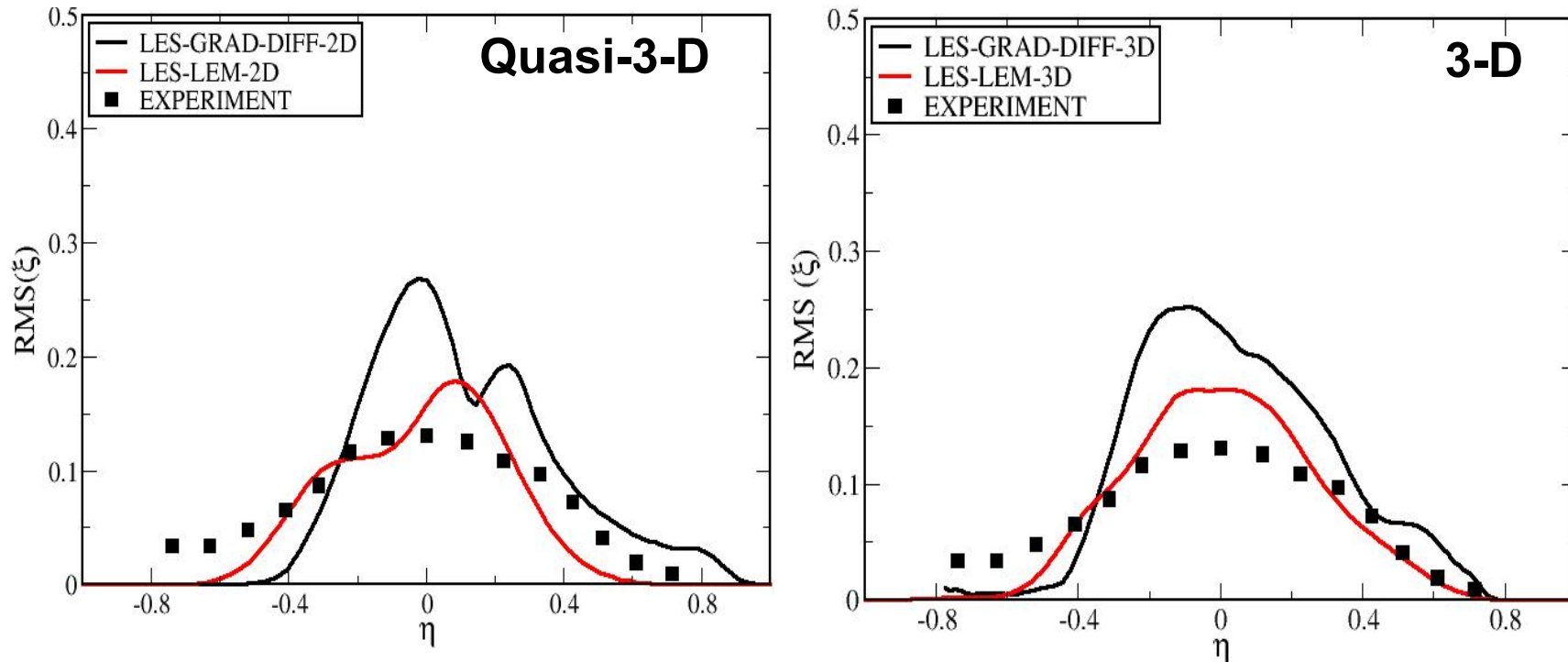


Mean Mixture Fraction, $Mc = 0.62$



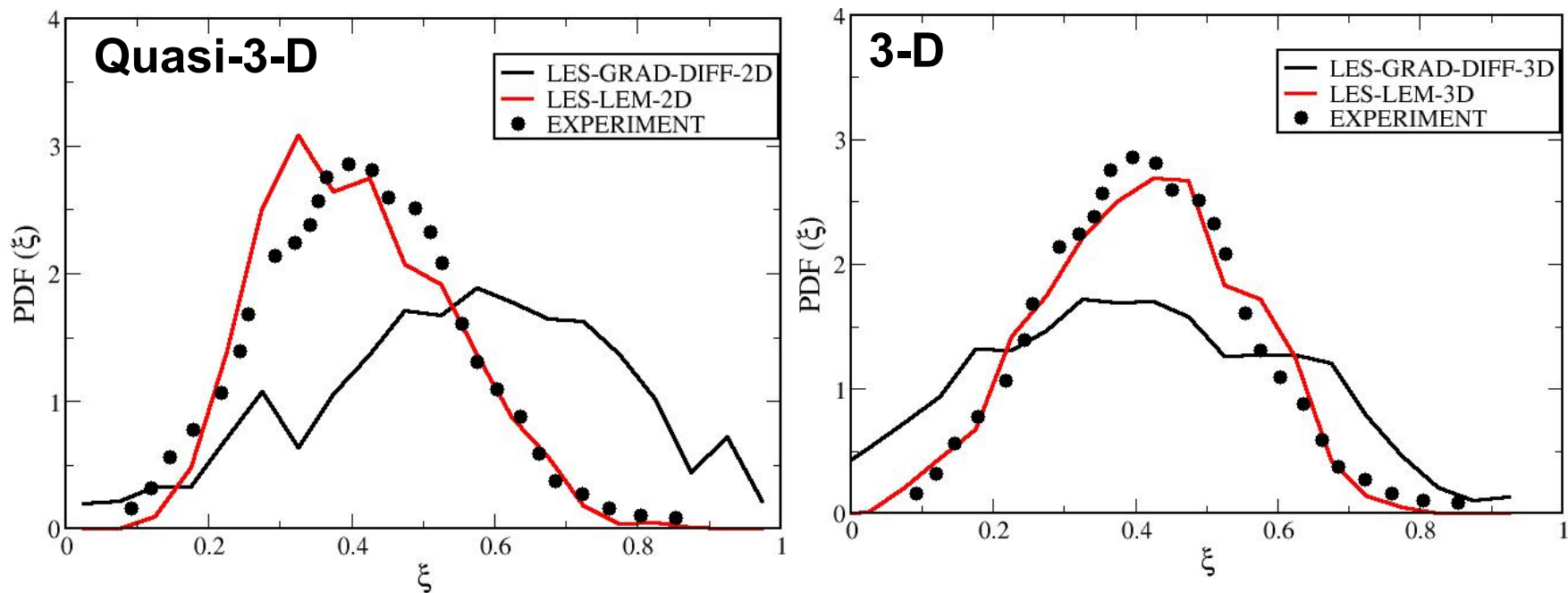
- Improvement in mean mixture fraction prediction for GRAD-DIFF
- LEMLES results are closer to the experiment near the edges

RMS of the Mixture Fraction, $Mc = 0.62$



- **GRAD-DIFF predicts a higher RMS compared to LEMLES**

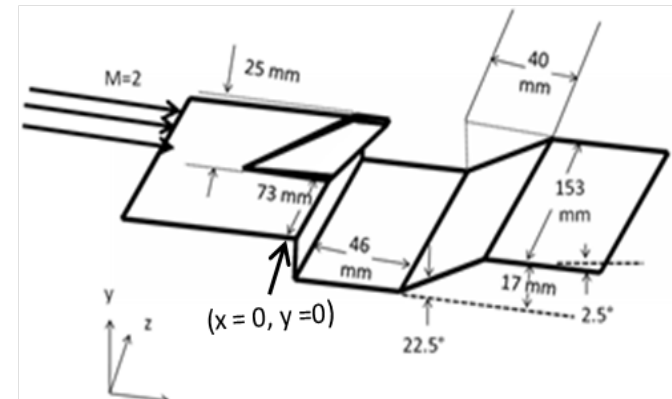
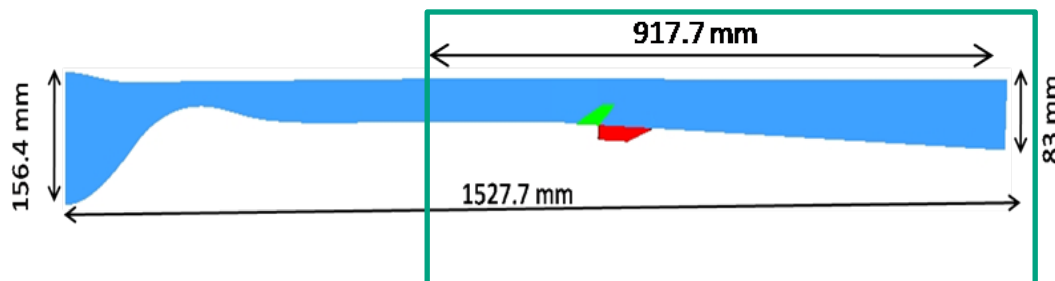
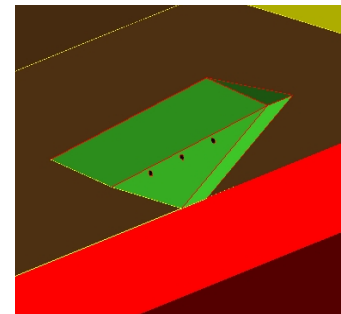
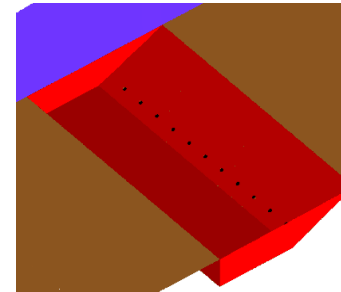
PDF of Mixture Fraction



- LEMLES correctly predicts the shape and width of the PDF
- Gradient diffusion LES fails to predict both these features
- Note: given PDF all scalar moments can be predicted

Numerical Studies of WPAFB TC19

- Full test facility is numerically simulated
- Hybrid VLES-LES in the isolator
- Two configurations studied
 - Cavity with 11 injectors on aft ramp
 - Strut upstream of cavity with 6 injectors
- 8+ million cells, 12/18 LEM cells per LES cell
 - Smallest mesh size ~ 0.01 mm
 - 30+ points in wall boundary layer

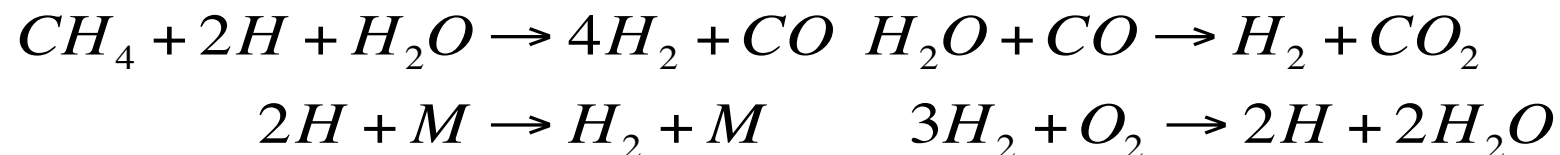


Mach 2 Flow Conditions

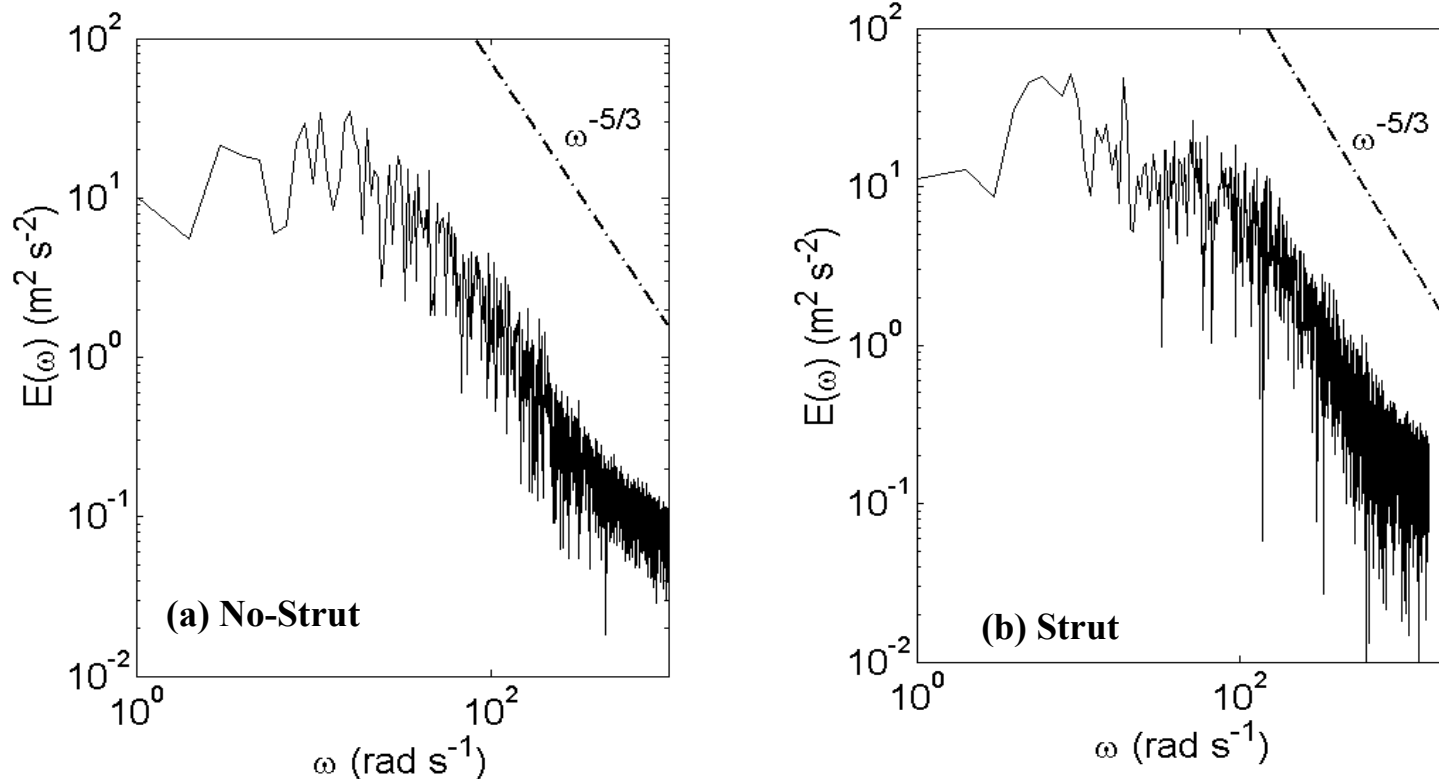
Test Configuration	Stagnation Conditions		Isentropic Conditions at Isolator					
	P0 (kPa)	T0 (K)			\dot{m}_{air} (kg/s)	\dot{m}_{water} (kg/s)	Relative Humidity (%)	Fuel Rate (SLPM)
No-Strut	414	589	53	327	3.2	25	4	225
Strut (Reacting)	207 (449)	564 (600)	26	313	1.6	25	9	270

- CH₄ – H₂ blended fuel (70% - 30%)
- Reduced 4-step, 8 species mechanisms*
- Local Reynolds number, Re_x ~ 42e6 m⁻¹
- Stagnation conditions for strut reacting case are changed (shown in red)

P _{-inj} (KPa)	T _{-inj} (K)
690	300



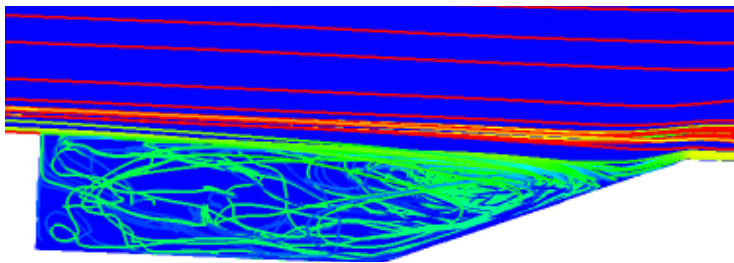
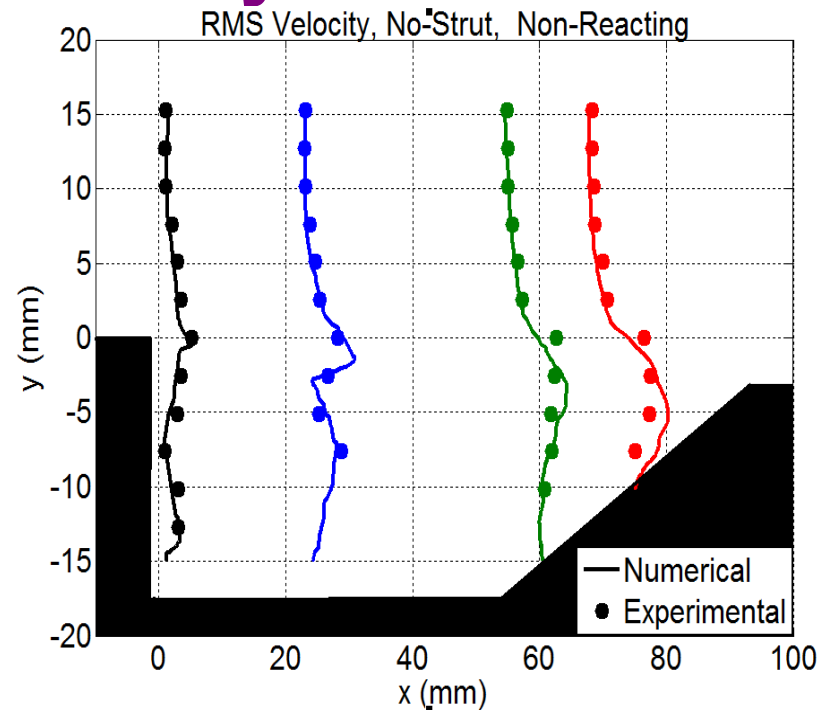
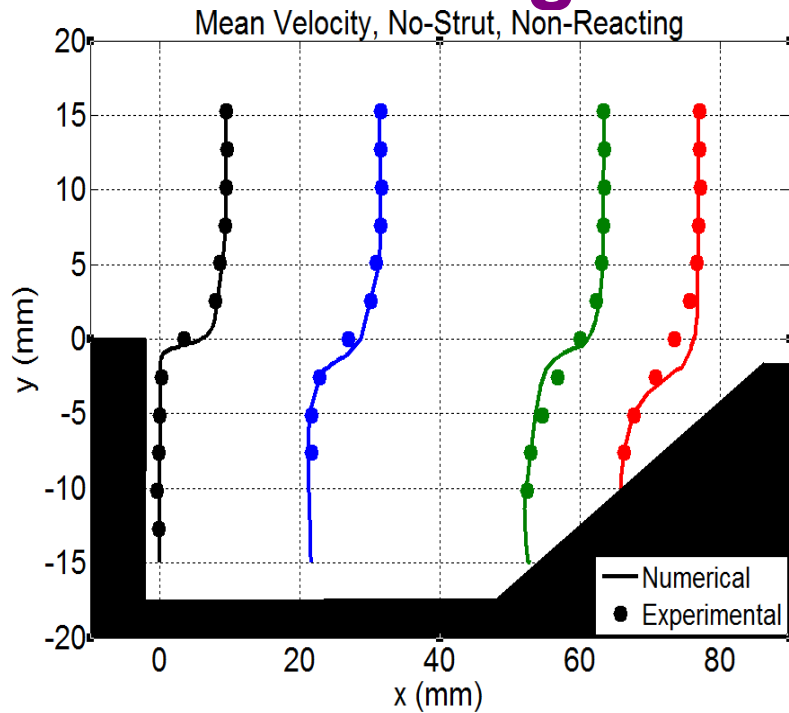
Energy Spectrum



(a) No-Strut: $X = 53.8$ mm within the shear layer and (b) Strut: $X = 33.8$ mm in the strut wake

Recover $k^{-5/3}$ law in shear layer and strut wake

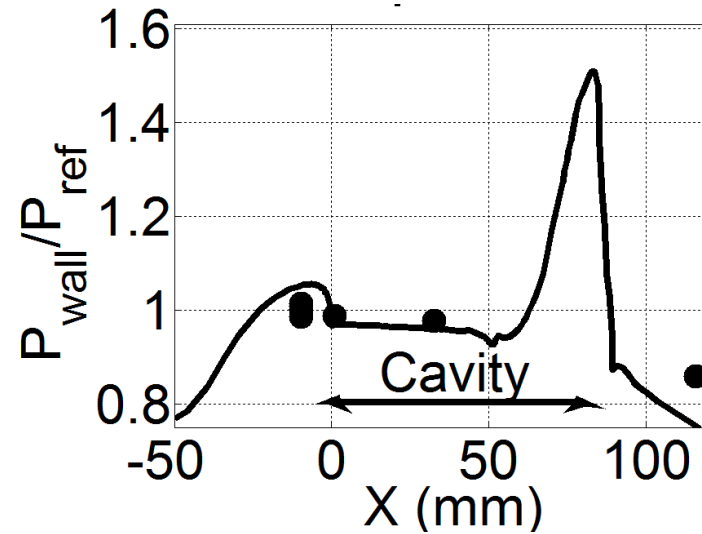
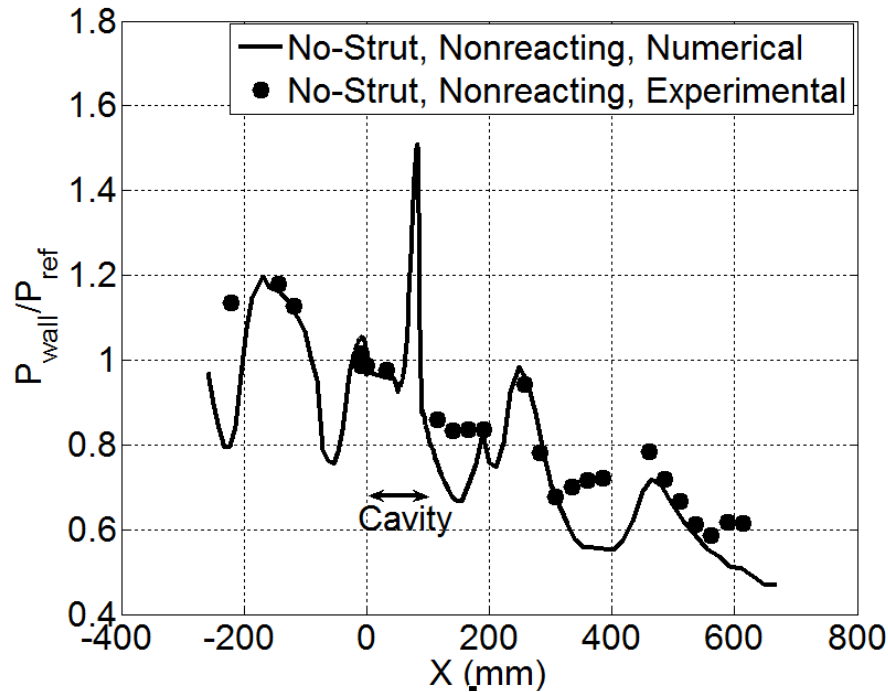
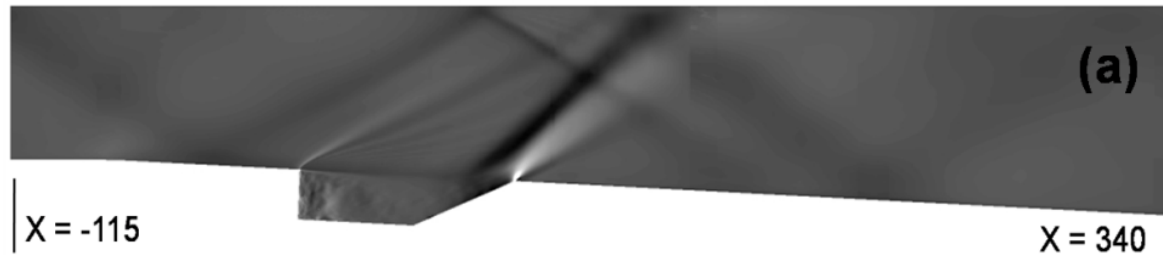
Non-reacting Flow: Cavity without Strut



Time averaged mean velocity streamlines

- Large vortical flow inside cavity
- Good agreement with data

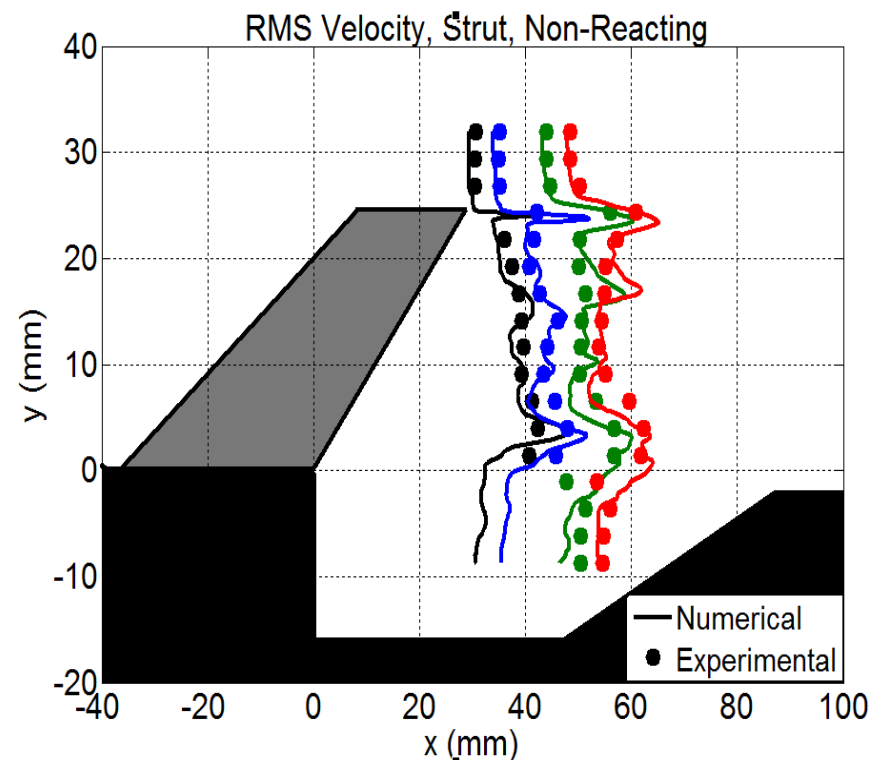
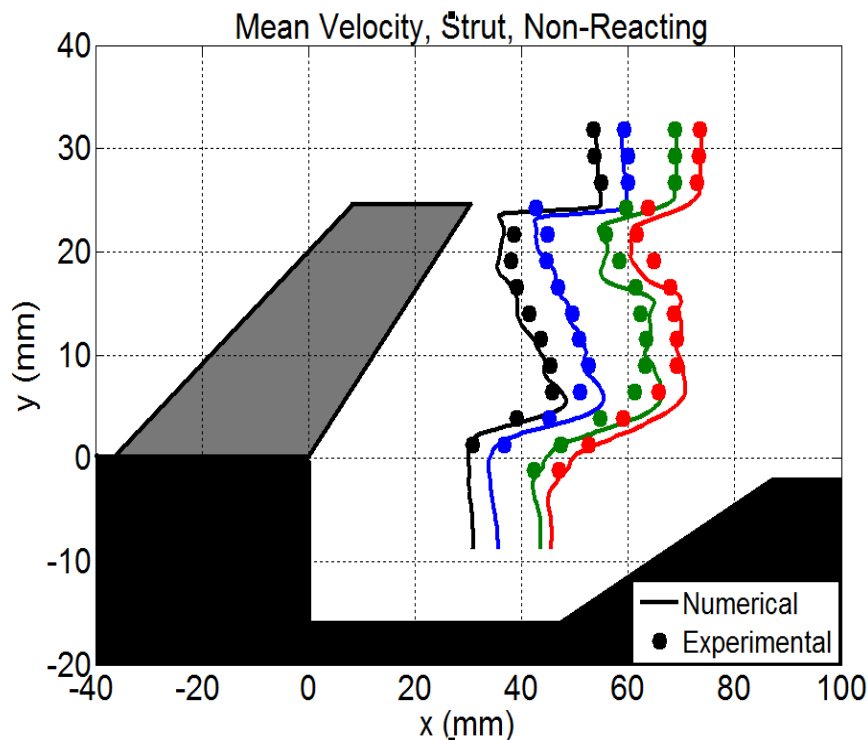
Wall Pressure



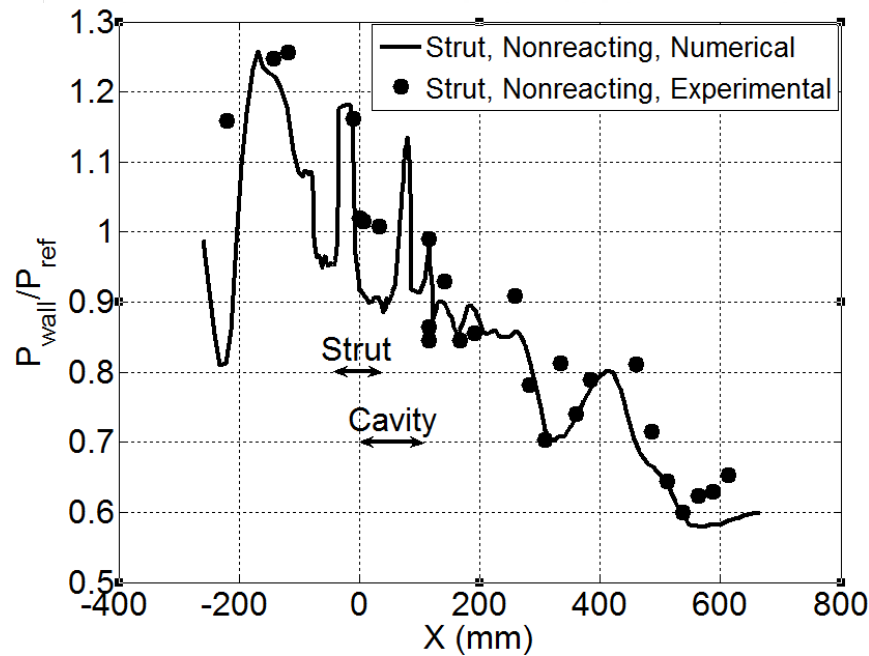
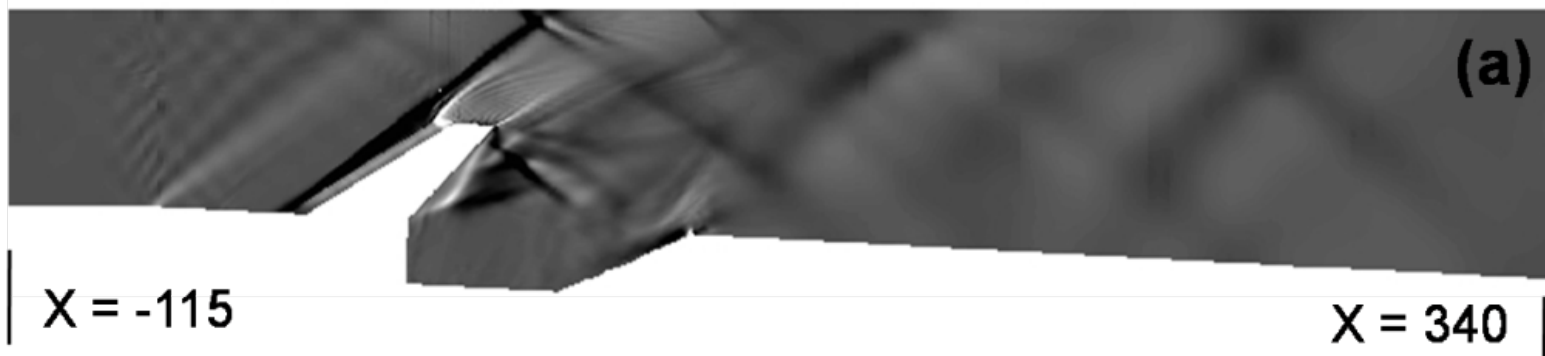
Wall Pressure

Velocity Comparisons: Non-Reacting, Strut

- Multiple shear layers in the wake of strut
- The overall spreading of wall-bounded cavity shear layer and velocity fluctuations are captured reasonably well



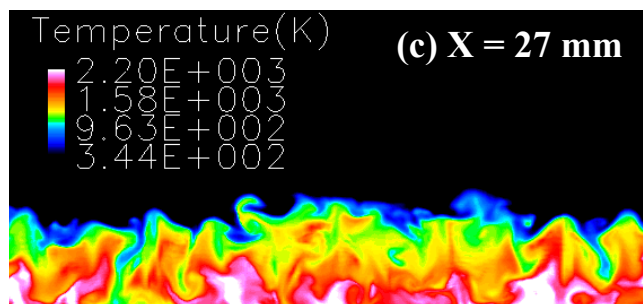
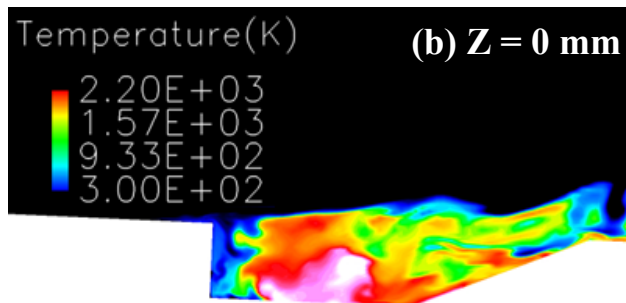
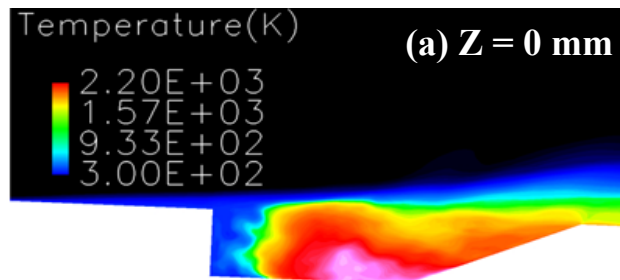
Wall Pressure Strut



- Shock off strut LE (X= -30 mm)
- re-compression shock at aft ramp portion (X ~ 86 mm)
- Expansion at the cavity leading edge
- Expansion-compression around strut top edge

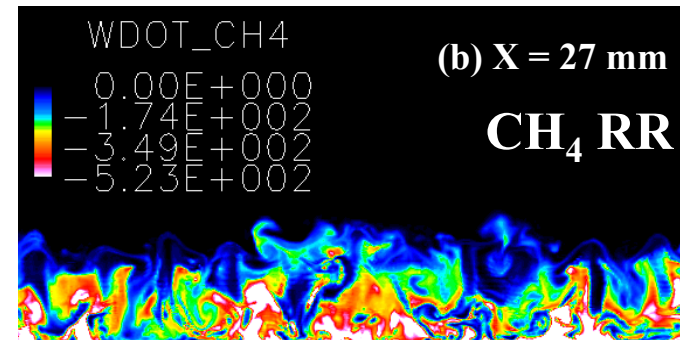
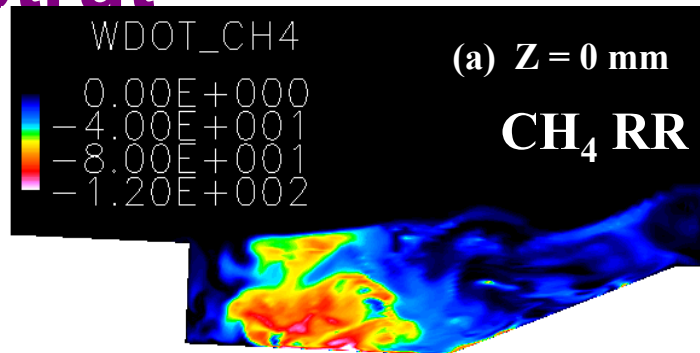
Wall Pressure

Reacting Case: Temperature Field No-Strut

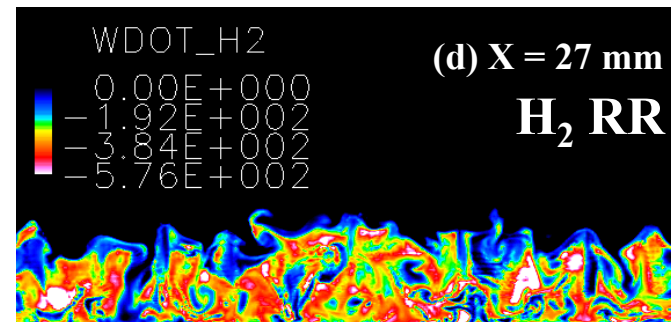
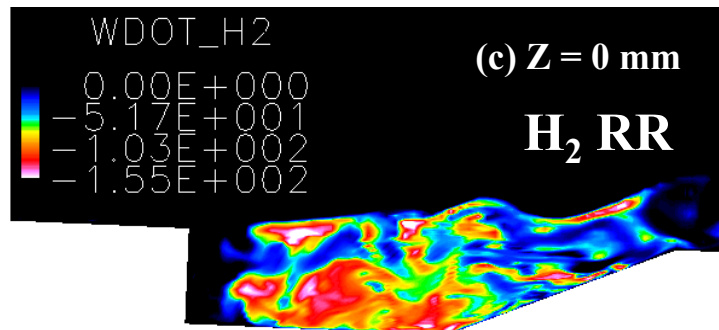


- Mean $\langle T \rangle$ shows that the cavity is full of products
 - Lifts shear layer for oxidizer entrainment into the cavity
- Instantaneous Temperature shows more variation in the cavity
- T at span-wise location (X = 27 mm) shows significant 3D structures
- High level of turbulence generated by aft wall fuelling

Flame Structure and Reaction Rate: No-Strut



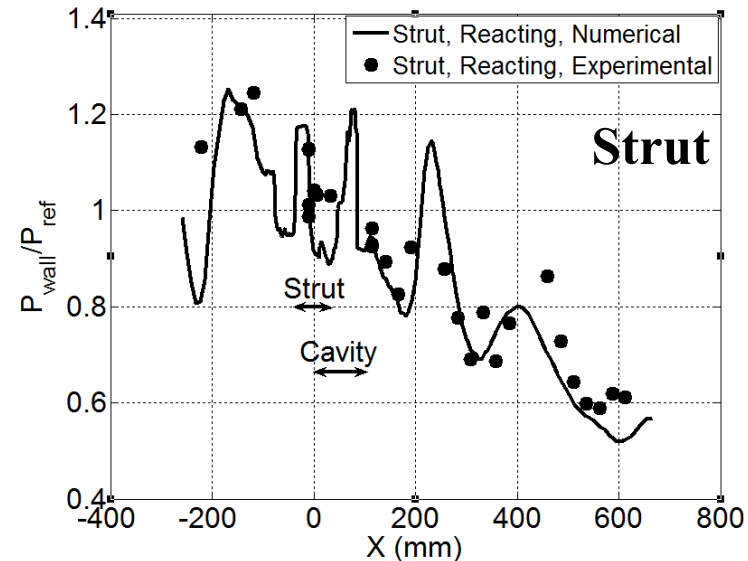
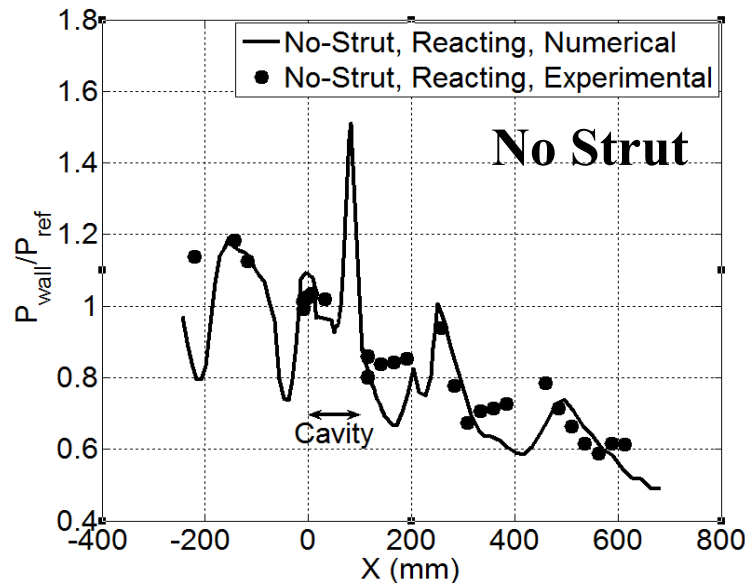
Reaction rate of CH₄ at (a) Z = 0 plane and (b) X = 27 mm span-wise plane



Reaction rate of H₂ at (c) Z = 0 plane and (d) X = 27 mm span-wise plane

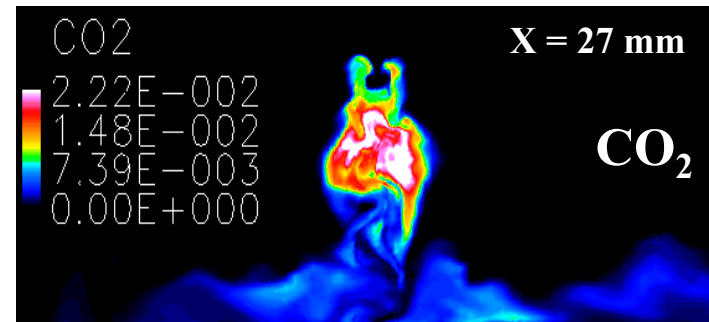
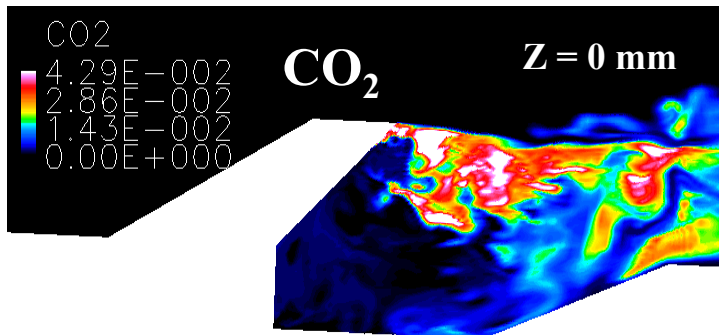
Methane and Hydrogen flame structures

Reacting Case: with and without Strut

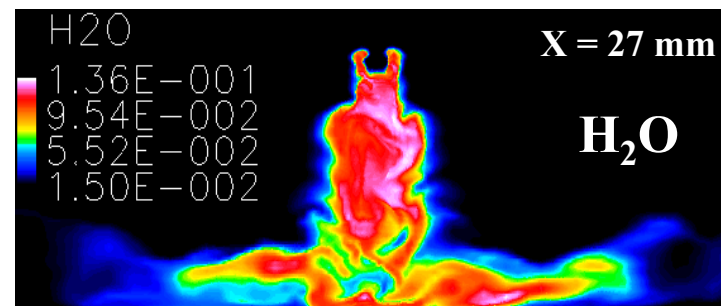
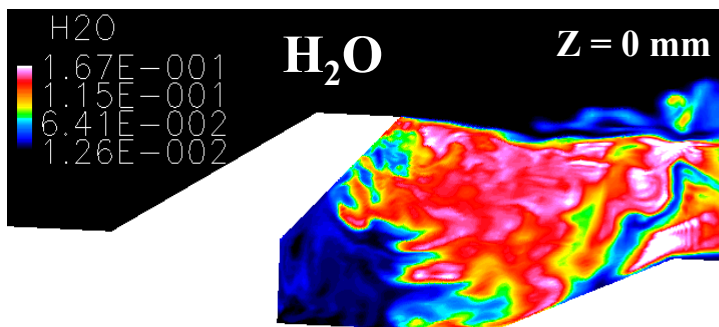


- Pressure comparison shows some reasonable agreement
- Peaks observed at locations where there are no pressure data locations of secondary shocks

Instantaneous Contours of Products: Strut

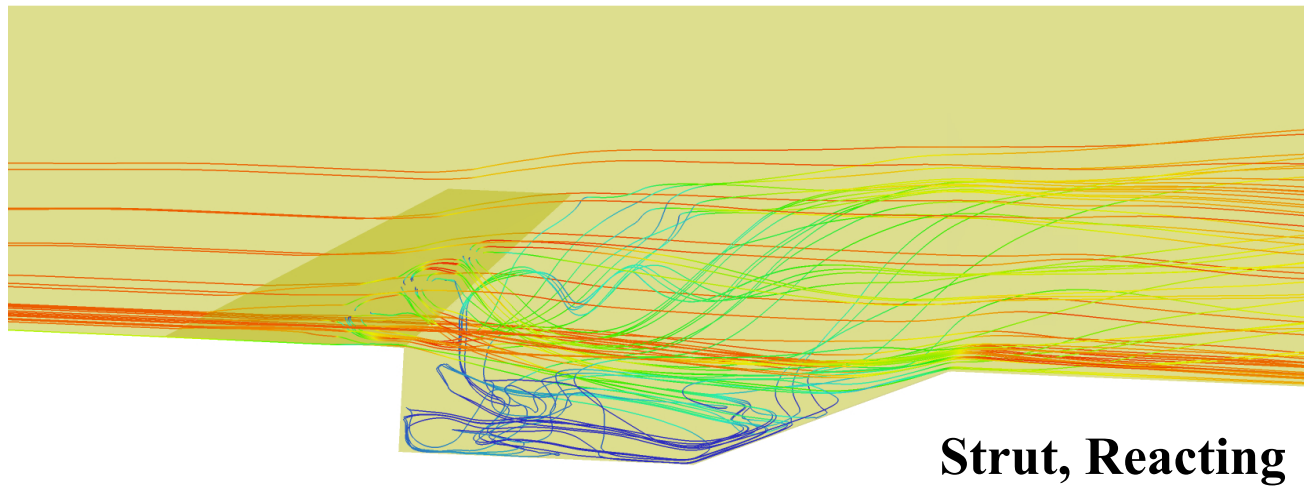
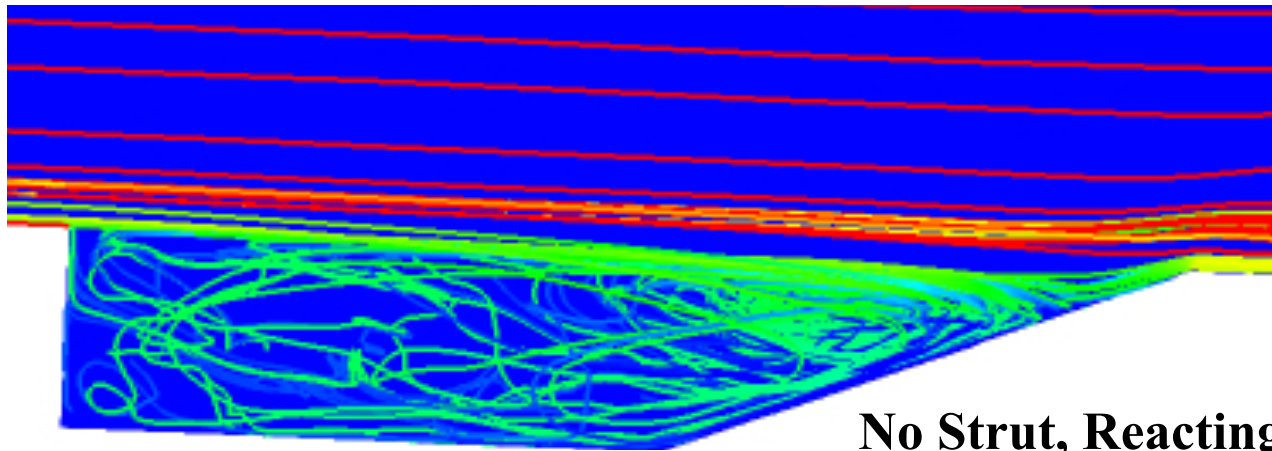


Contours of CO₂ at (a) Z = 0 plane and (b) X = 27 mm span-wise plane

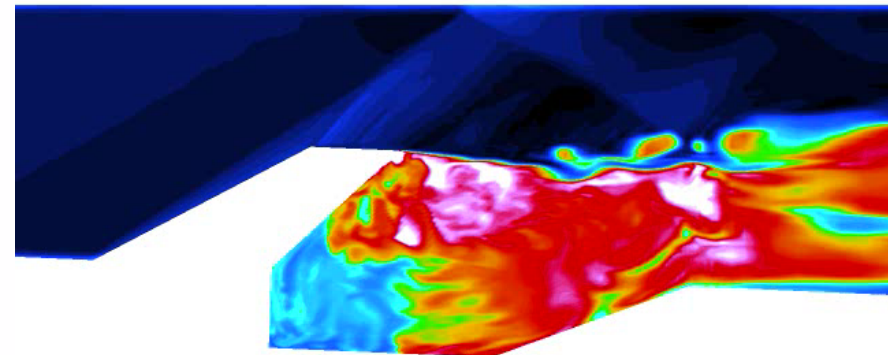
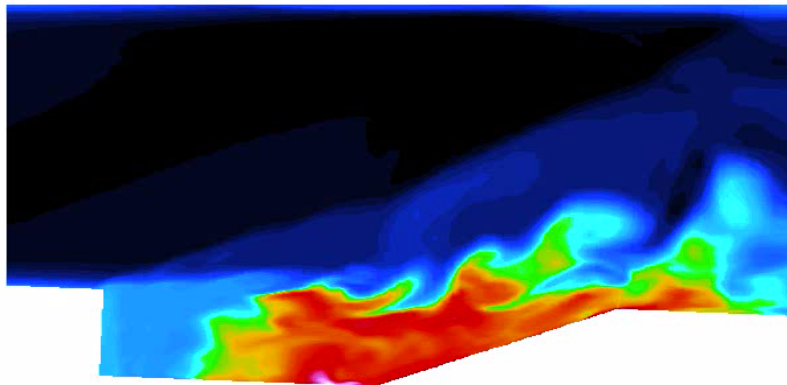
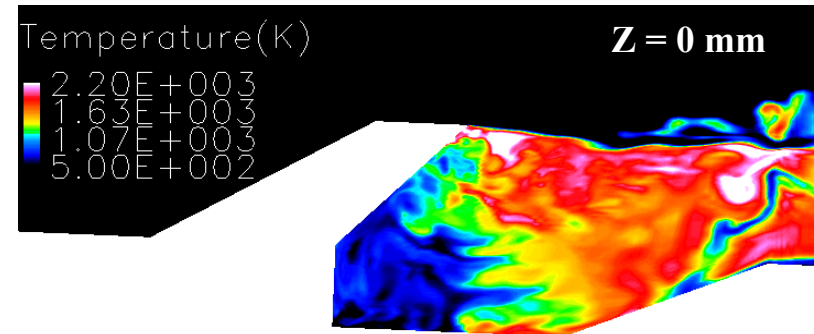
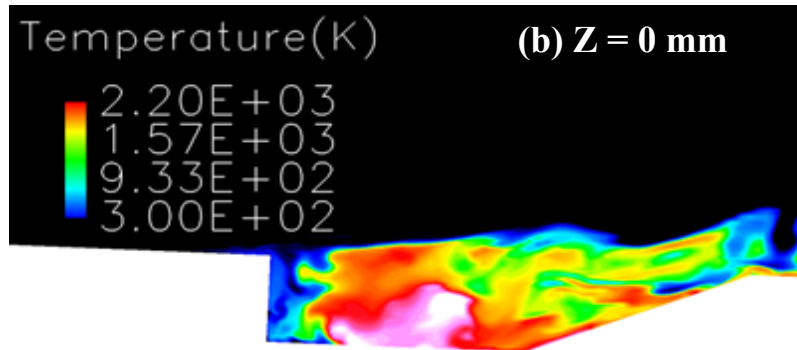


Contours of H₂O at (a) Z = 0 plane and (b) X = 27 mm span-wise plane

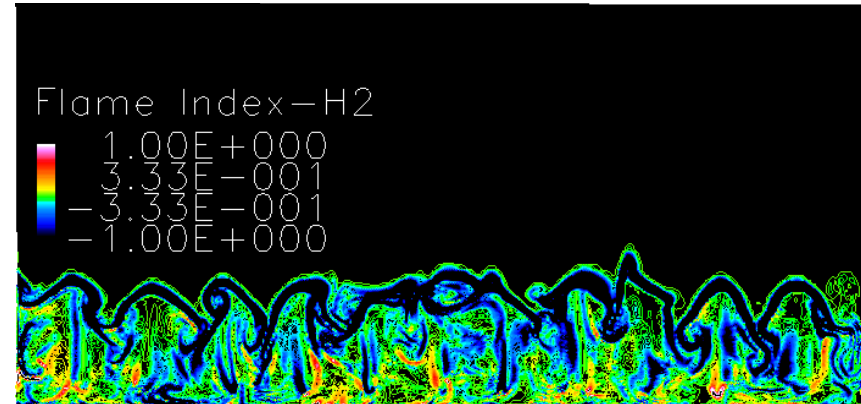
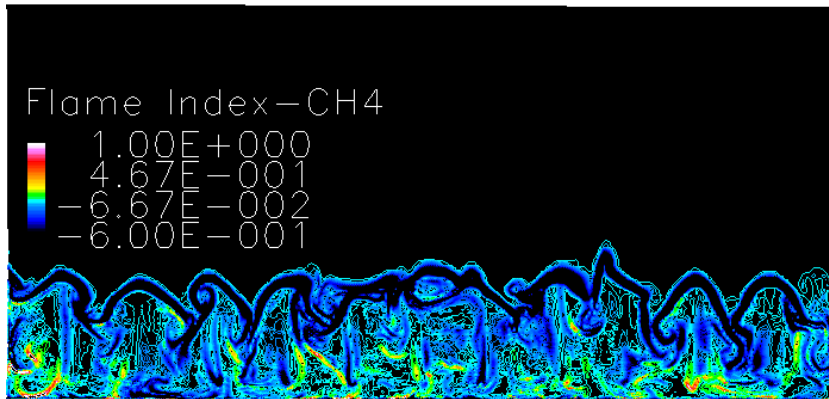
Streamlines



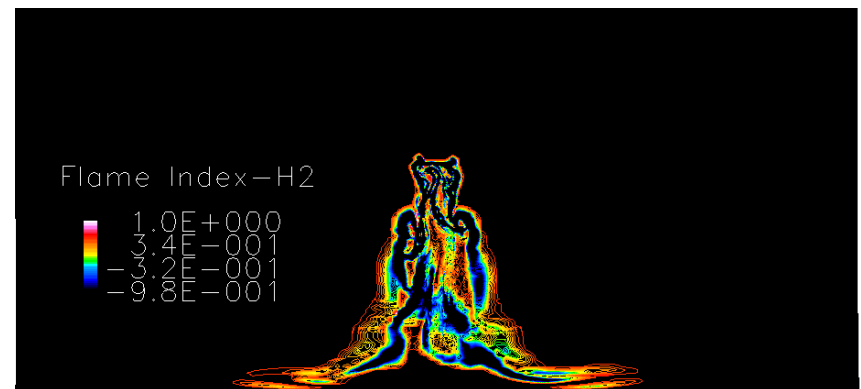
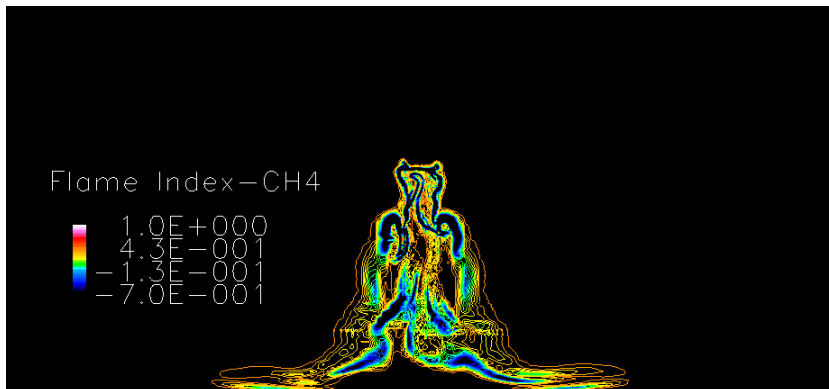
Strut and No-Strut Comparison



$$\text{Flame Index} = \text{normalized } \nabla Y_F \cdot \nabla Y_O$$



No Strut – Cavity



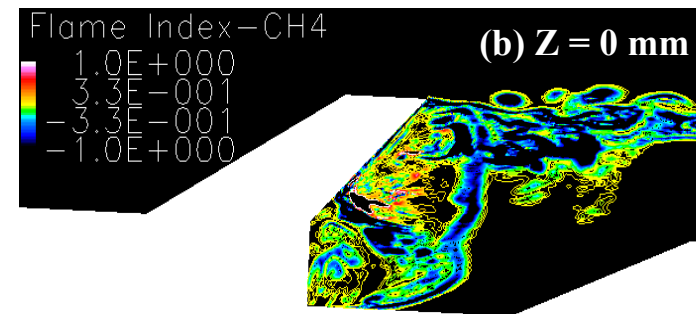
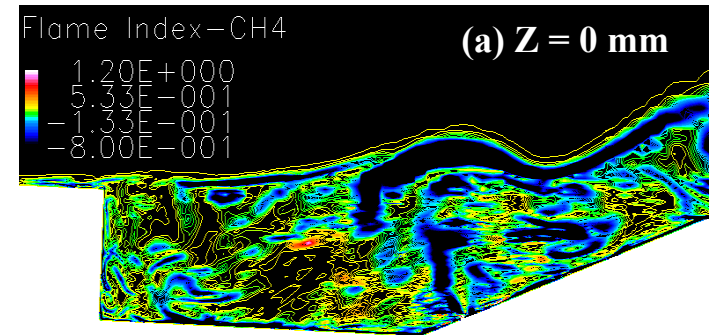
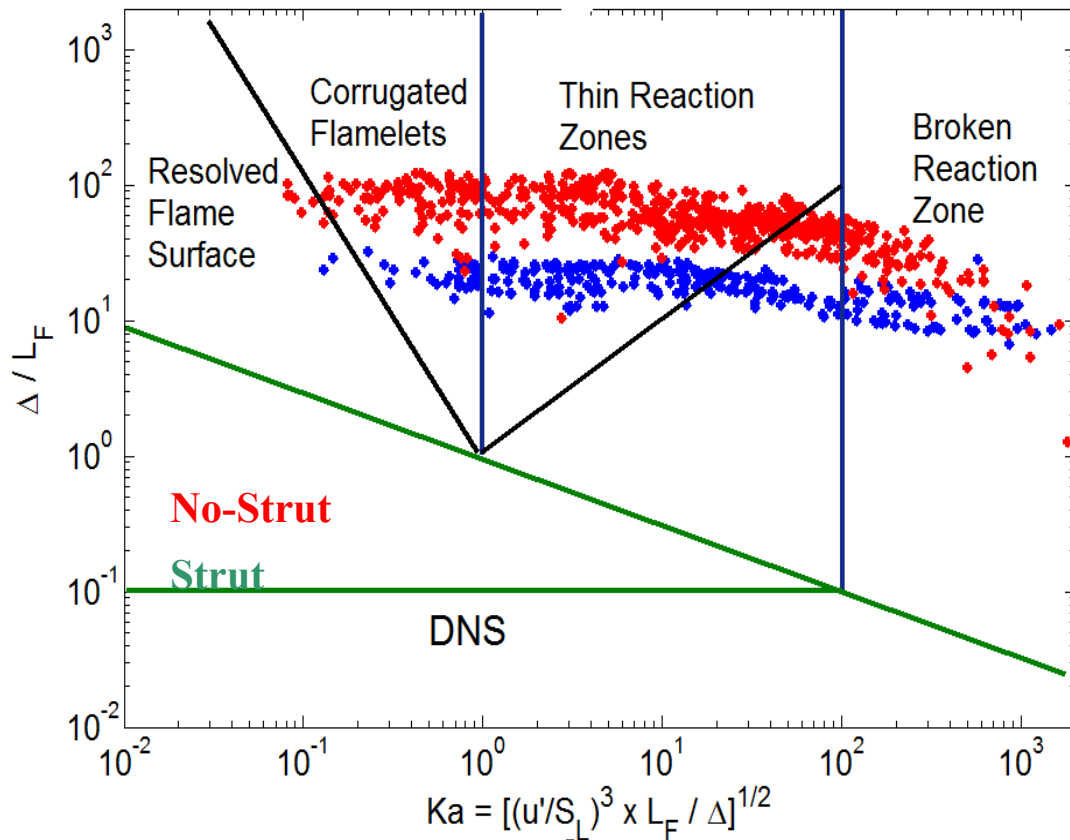
CH₄

Strut Wake

H₂

- **Flame Index > 0 for premixed flame and < 0 for diffusion flame**
Day 2, Lecture 8, Suresh Menon, Georgia Tech

Reacting Flow – Flame Regimes for LES



Flame Index for CH₄

- Strong variation of Ka from flamelet to broken reaction zone
- LEMLES captures all regimes without model change

Chemical Kinetics Modeling

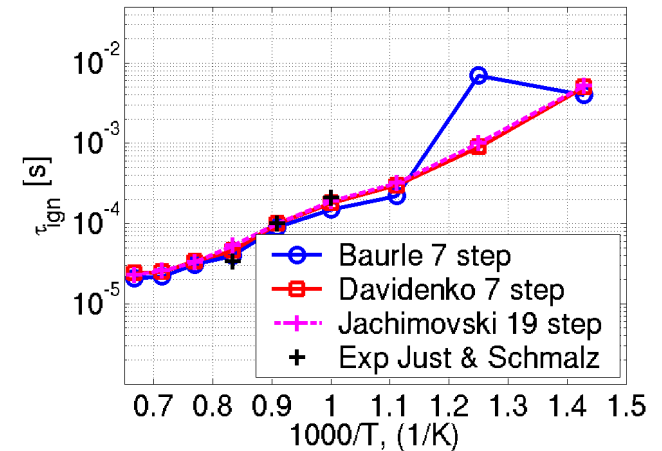
The description of the chemical kinetics is very important as its time-scales (τ_c) are on the same order-of-magnitude as those of the flow ($\tau_I, \tau_\Delta, \tau_T$).

Parameters for the 9 species, 19-step Jachimowski mech.

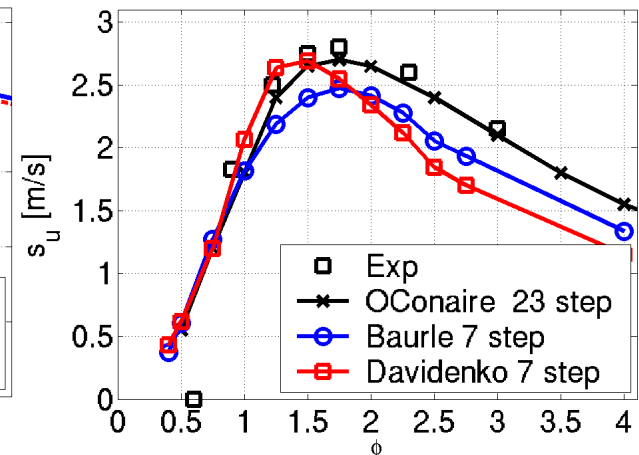
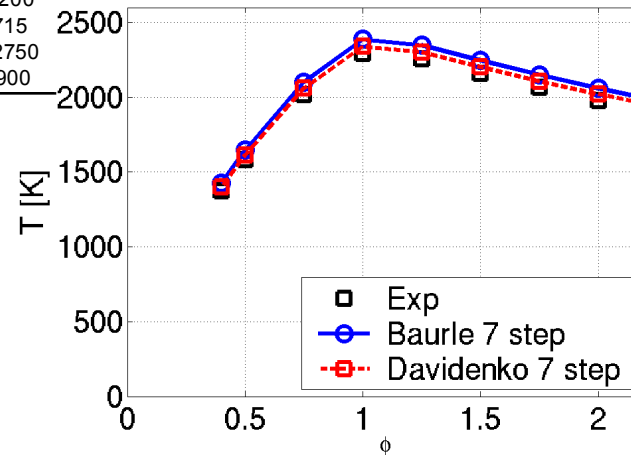
Reaction	A [mol-cm-s]	n	T _a [K]
H ₂ +O ₂ ⇌HO ₂ +H	1.0·10 ¹⁴	0	28000
H+O ₂ ⇌OH+O	2.6·10 ¹⁴	0	8400
O+H ₂ ⇌OH+H	1.8·10 ¹⁰	1.0	4450
OH+H ₂ ⇌H ₂ O+H	2.2·10 ¹³	0	2575
OH+OH⇌H ₂ O+O	6.3·10 ¹²	0	545
H+OH+M⇌H ₂ O+M	2.2·10 ²²	-2.0	0
2H+M⇌H ₂ +M	6.4·10 ¹⁷	-1.0	0
H+O+M⇌OH+M	6.0·10 ¹⁶	-0.6	0
H+O ₂ +M⇌HO ₂ +M	2.1·10 ¹⁵	0	-500
HO ₂ +H⇌OH+OH	1.4·10 ¹⁴	0	540
HO ₂ +H⇌H ₂ O+O	1.0·10 ¹³	0	540
HO ₂ +O⇌O ₂ +OH	1.5·10 ¹³	0	475
HO ₂ +OH⇌H ₂ O+O ₂	8.0·10 ¹²	0	0
HO ₂ +HO ₂ ⇌H ₂ O ₂ +O ₂	2.0·10 ¹²	0	0
H+H ₂ O ₂ ⇌OH+HO ₂	1.4·10 ¹²	0	1800
O+H ₂ O ₂ ⇌H ₂ +HO ₂	1.4·10 ¹⁴	0	3200
OH+H ₂ O ₂ ⇌H ₂ O+HO ₂	6.1·10 ¹²	0	715
H ₂ O ₂ +M⇌2OH+M	1.2·10 ¹⁷	0	22750
O+O+M⇌O ₂ +M	6.0·10 ¹³	0	-900

Parameters for the 6 species, 7-step Davidenko mech.

Reaction	A [mol-cm-s]	n	T _a [K]
H ₂ +O ₂ ⇌OH+OH	1.700·10 ¹³	0	24044
H+O ₂ ⇌OH+O	1.987·10 ¹⁴	0	8456
OH+H ₂ ⇌H ₂ O+H	1.024·10 ⁸	1.60	1660
O+H ₂ ⇌OH+H	5.119·10 ⁴	2.67	3163
OH+OH⇌H ₂ O+O	1.506·10 ⁹	1.14	50
H+OH+M⇌H ₂ O+M	2.212·10 ²²	-2.00	0
H+H+M⇌H ₂ +M	9.791·10 ¹⁶	-0.60	0



- 1 step Marinov *et al*
- 2 step Rogers & Chintz
- 7 step Davidenko *et al*
- 7 step Baurle & Girimaji
- 14 step Zhukov?

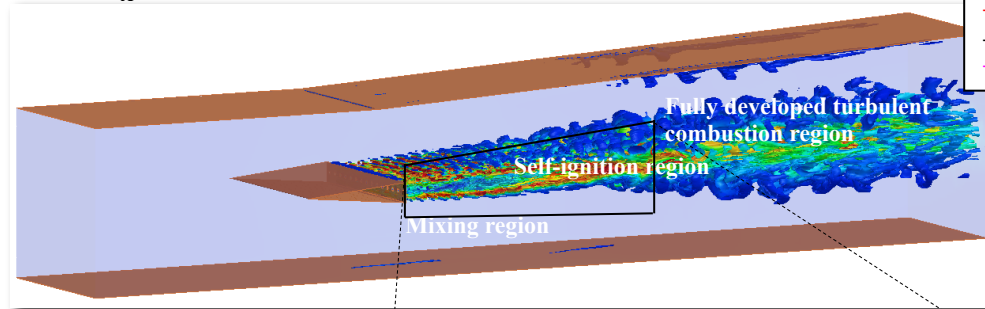
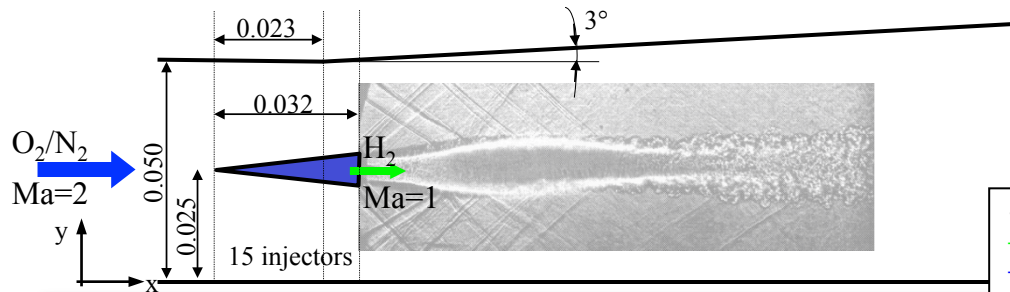


The Waidmann *et al* Combustor

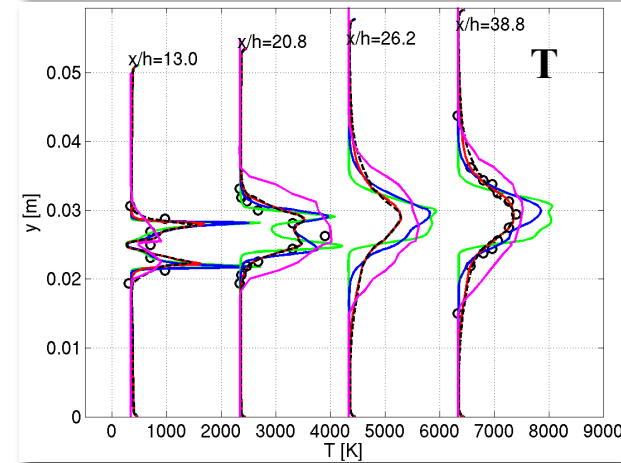
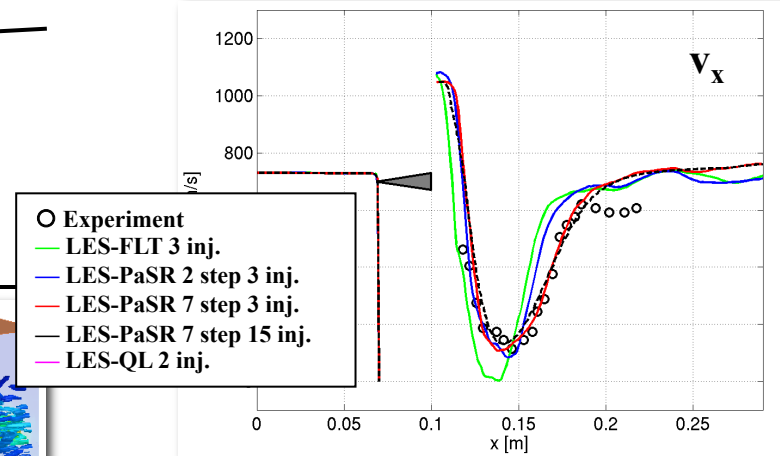
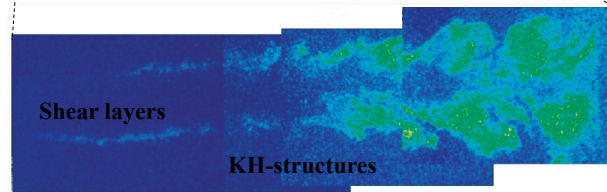
DLR experimental investigation

Waidmann W., Brummund U. & Nuding J.; 1995, 8th Int. Symp. on Transp. Phenom. In Comb., p 1473.

Waidmann W., Alff F., Brummund U., Böhm M., Clauss W. & Oswald M.; 1995, Space Tech. 15, p 421.



Fureby *et al*
 3 injectors, 5 Mcells
 15 injectors, 25 Mcells
Génin & Menon
 2 injectors, 2.5 Mcells



Berglund & Fureby, 2006, 31st Int. Symp. On Comb.

Génin & Menon, 2009, AIAA 2009-0132

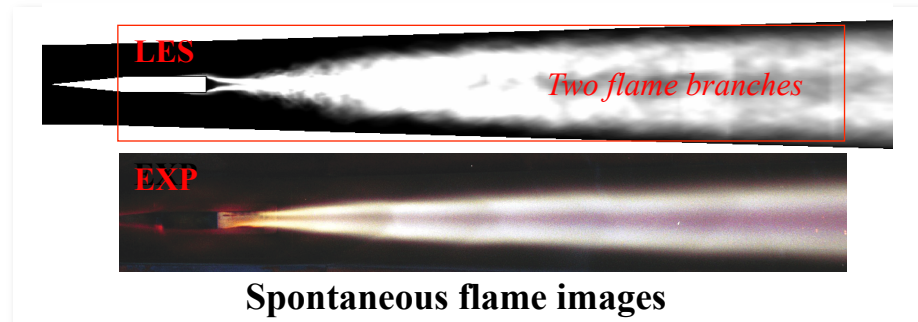
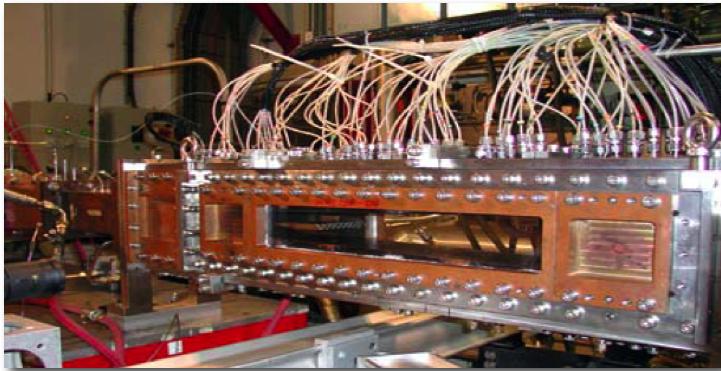
Fureby *et al.*, 2011, 28th ISSW, Manchester

Sunami-Magré Combustor

Joint ONERA / JAXA experimental (scramjet) combustor study

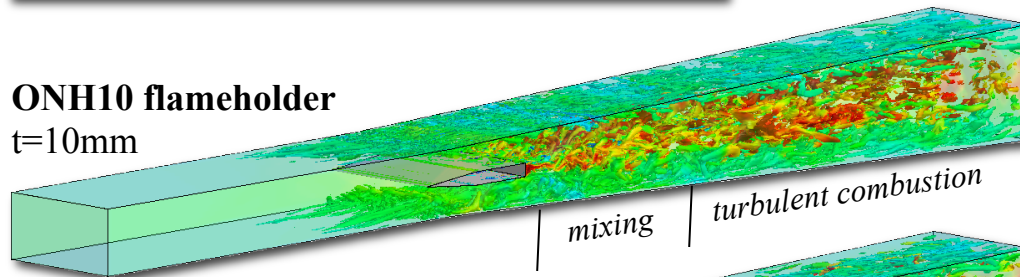
Sunami T., Murakami A., Kudo K., Kodera M. & Nishioka M., AIAA 2002-5116,

Sunami T., Magré P., Bresson A., Grisch F., Orain M., & Kodera M., AIAA 2005-3304



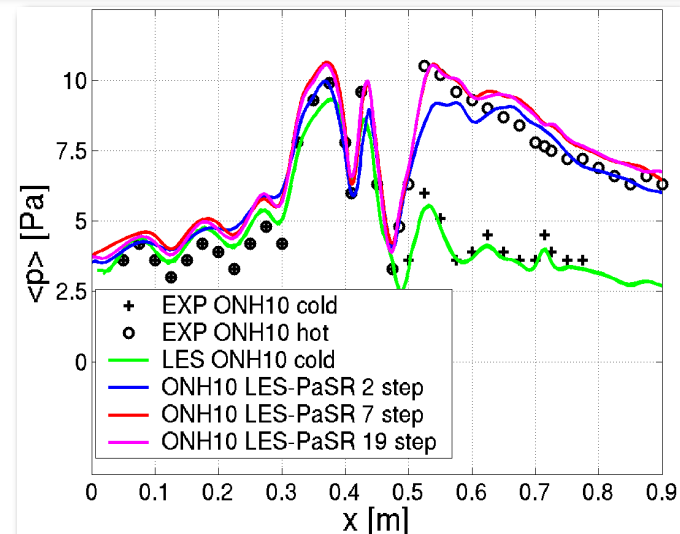
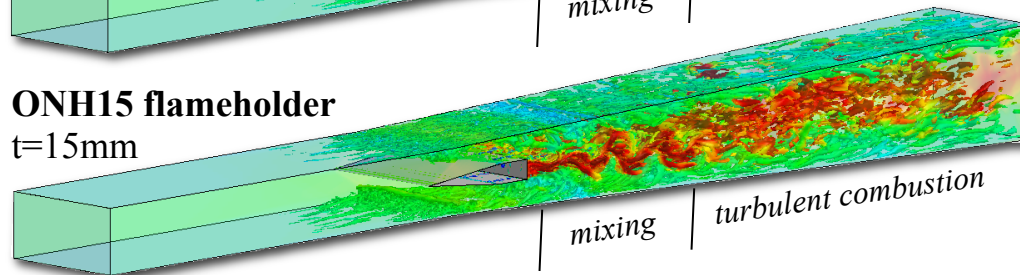
ONH10 flameholder

$t=10\text{mm}$



ONH15 flameholder

$t=15\text{mm}$

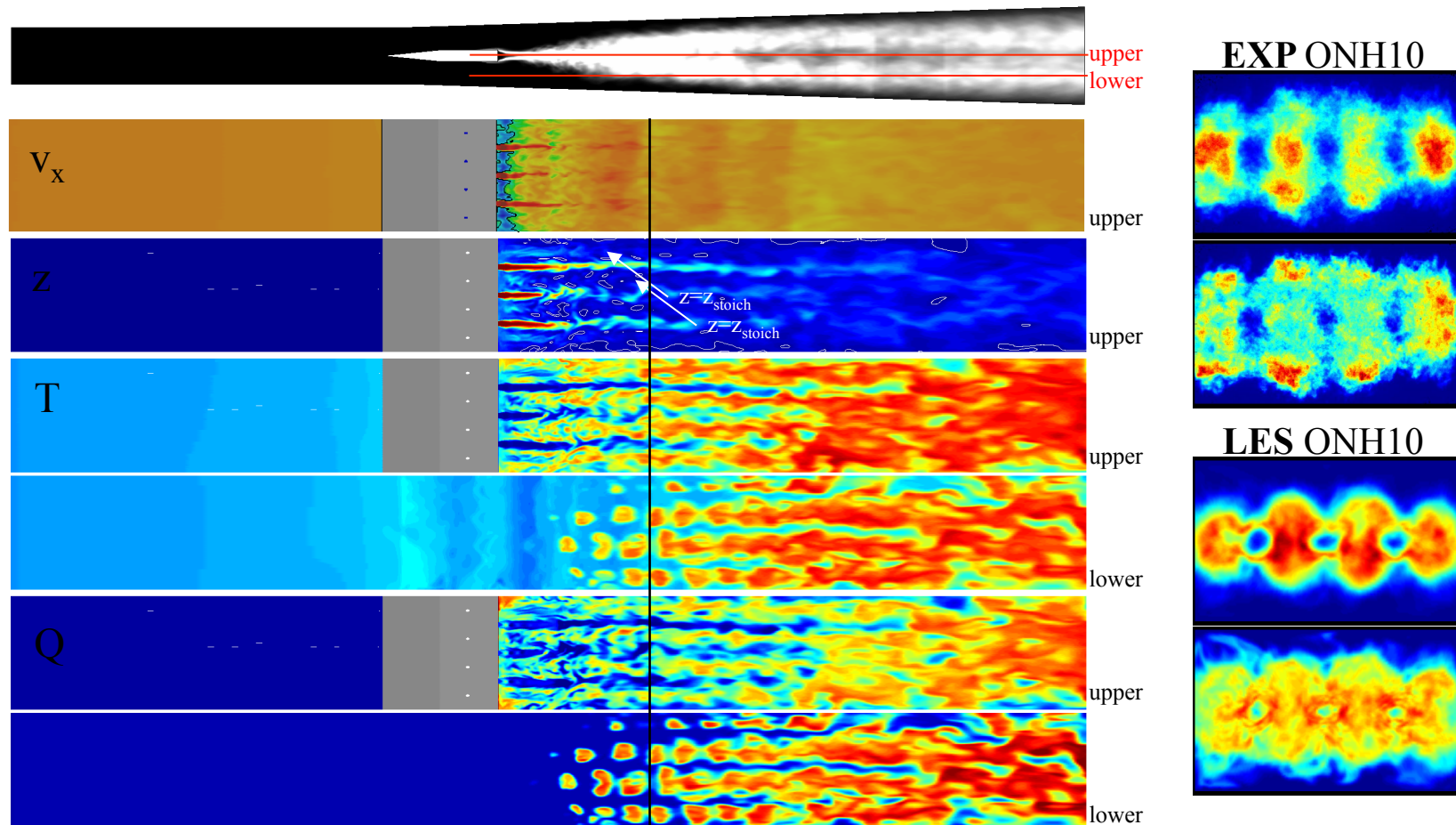


Berglund *et al*, 2009, AIAA J.
Sabelnikov & Fureby In Preparation 2011

ONERA

Sunami-Magré Combustor cont' d

Supersonic flame structure investigation (ONH10) and OH comparison

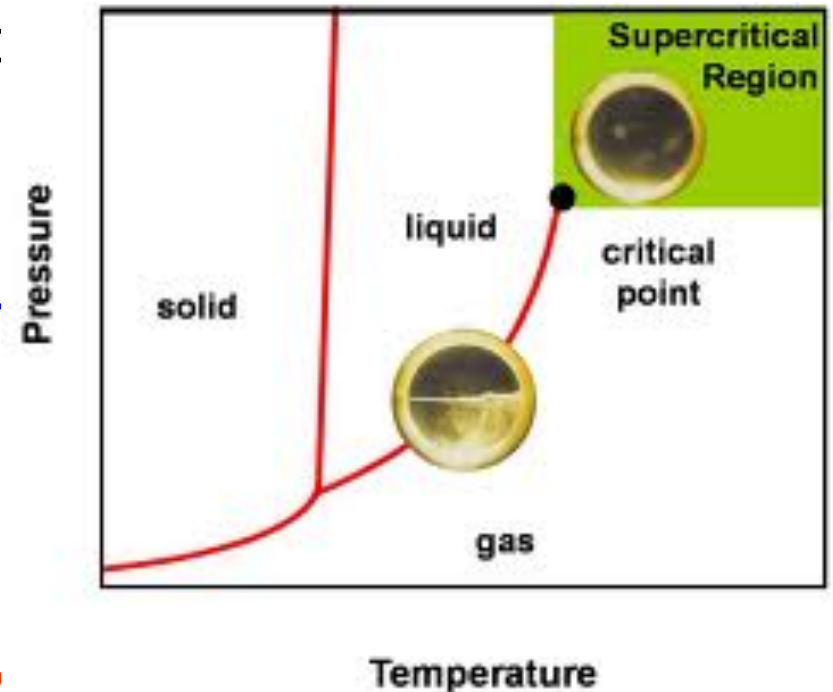


Berglund *et al*, 2009, AIAA J.
Sabelnikov & Fureby In Preparation 2011

ONERA

Real Gas: Basic thermodynamics

- Under atmospheric conditions, most fluids require a phase change to go from liquid to gas
 - Multiphase field: breakup, atomization, evaporation...
- Not necessary if $T > T_c$ OR $p > p_c$
 - Smooth interface
 - No surface tension
 - No latent heat of vaporization
- If $T > T_c$ AND $p > p_c$, fluid is supercritical



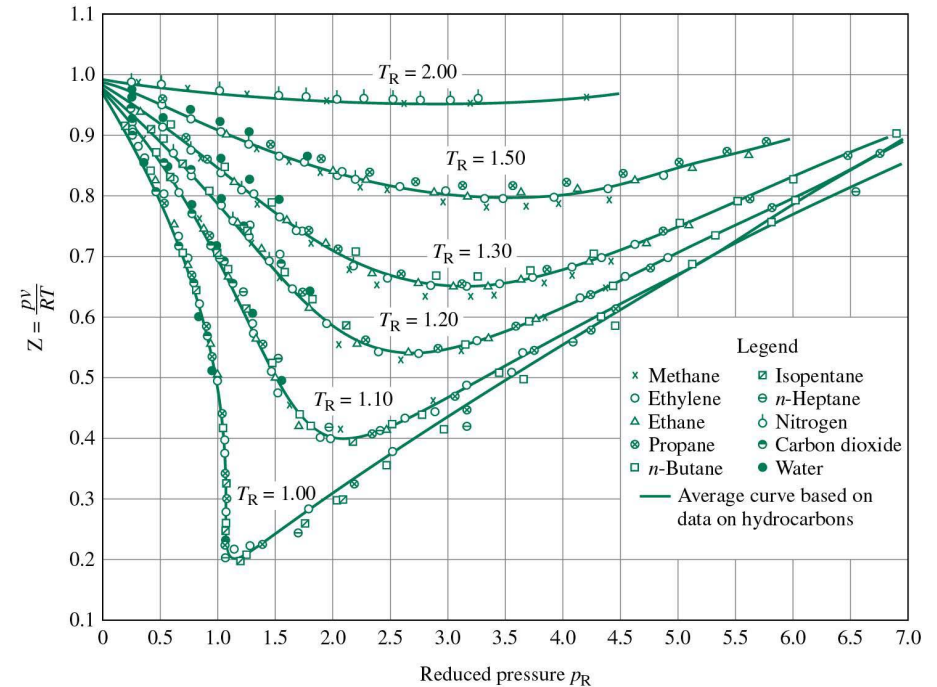
Basic thermodynamics

- A supercritical fluid may or may not follow the Ideal Gas Equation of State (IG EoS) $pV = R_u T$
- Departure from IG EoS caused by inter-molecular effects:
 - Molecules cannot be assumed to be points
 - Inter-molecular forces on top of simple collisions
- These real gas, i.e. non-ideal, effects occur when the density of the fluid is large enough
 - What is large enough?

Basic thermodynamics

$$Z = \frac{pV}{R_u T}$$

- Introduce compressibility
 - $Z=1 \Rightarrow$ ideal gas
 - $Z \neq 1 \Rightarrow$ real gas
- Hint at a universal behavior
 - Z is equivalent for simple species when normalizing T and p by T_c and p_c
- Mathematical translation into new EoS

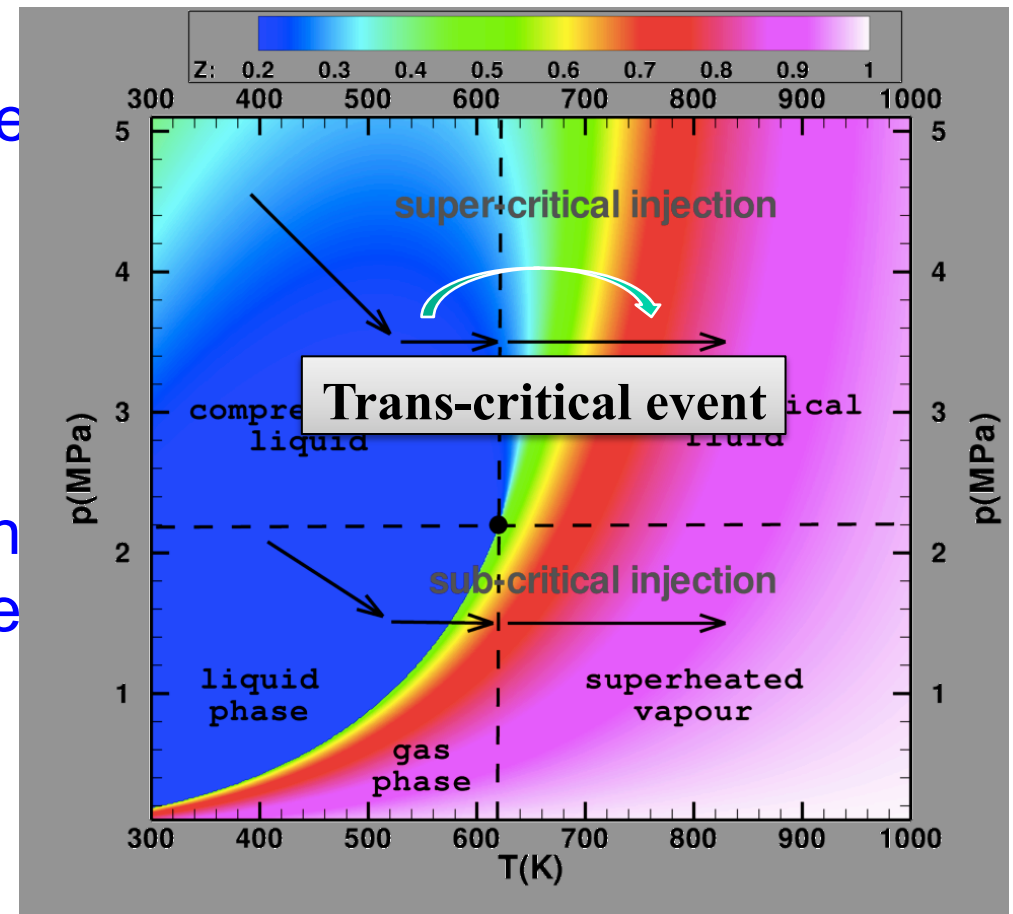


Relevance to combustion

- Overall trend is to increase pressure (GT, ICE, rockets)
- Three flows where real gas effects are important:
 - Sub-critical flows
 - All species gaseous, mild departures from $Z = 1$
 - Super-critical flows
 - Some species supercritical, $Z = 0.3$ to 1
 - Trans-critical flows
 - Some species are compressed liquids, Z can vary from 0.3 to 1 and pseudo-phase change phenomena

Relevance to combustion

- Concrete example: surrogate aircraft fuel
 - 82.6 % n-decane and 17.4% trimethylbenzene (Pitsch_2008a)
- Corresponding states principle (CSP)
 - Mixture behaves like a pure pseudo-fluid with pseudo critical properties



Issues for combustion modeling

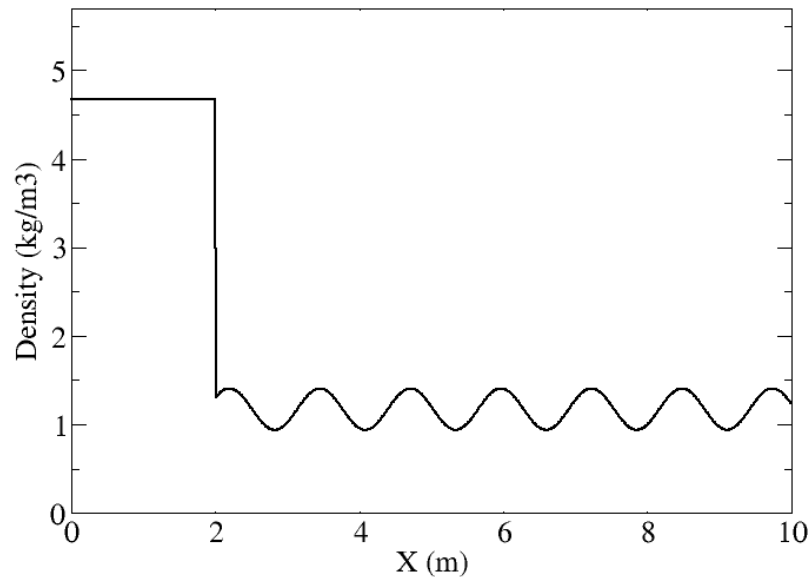
- Large density gradients
- Computational cost
- Additional unclosed terms
- Pressure dependence in reaction mechanisms
- EoS validity for a wide range of flow conditions and species must be understood and established
 - Cubic EoS such as Peng-Robinson (PR), Soave-Redlich-Kwong (SRK)
 - Higher order empirical EoS such as Benedict-Webb-Rubin (BWR)

Dealing with density gradients

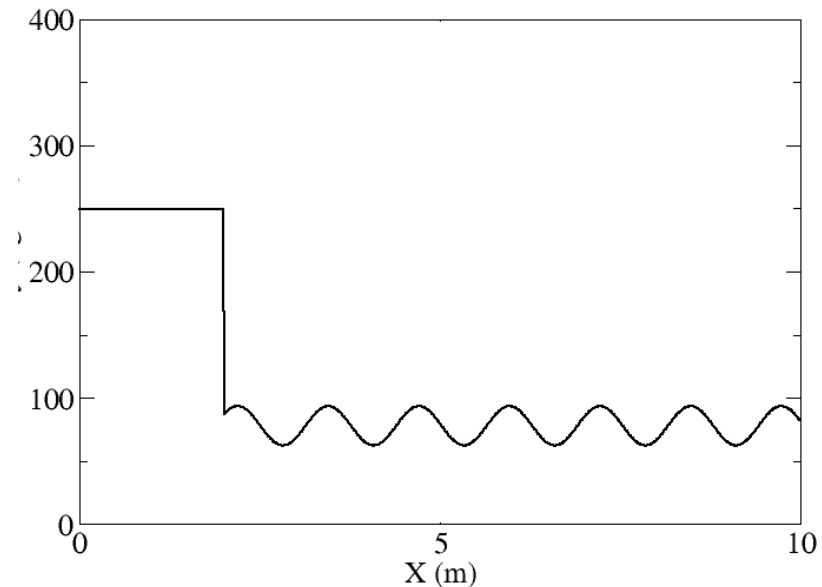
- Need to capture both large density gradients & turbulence at the same time
- Implement within the real gas EoS:
 - TVD MUSCL scheme using approximate Riemann solver for 3rd order accuracy
 - Dynamic switch based on local density gradients
- Pure central schemes cannot handle these gradients without huge resolution requirements
- Pure upwind schemes are too dissipative

Dealing with density gradients

- Shu-Osher test



Standard air $Z = 1$

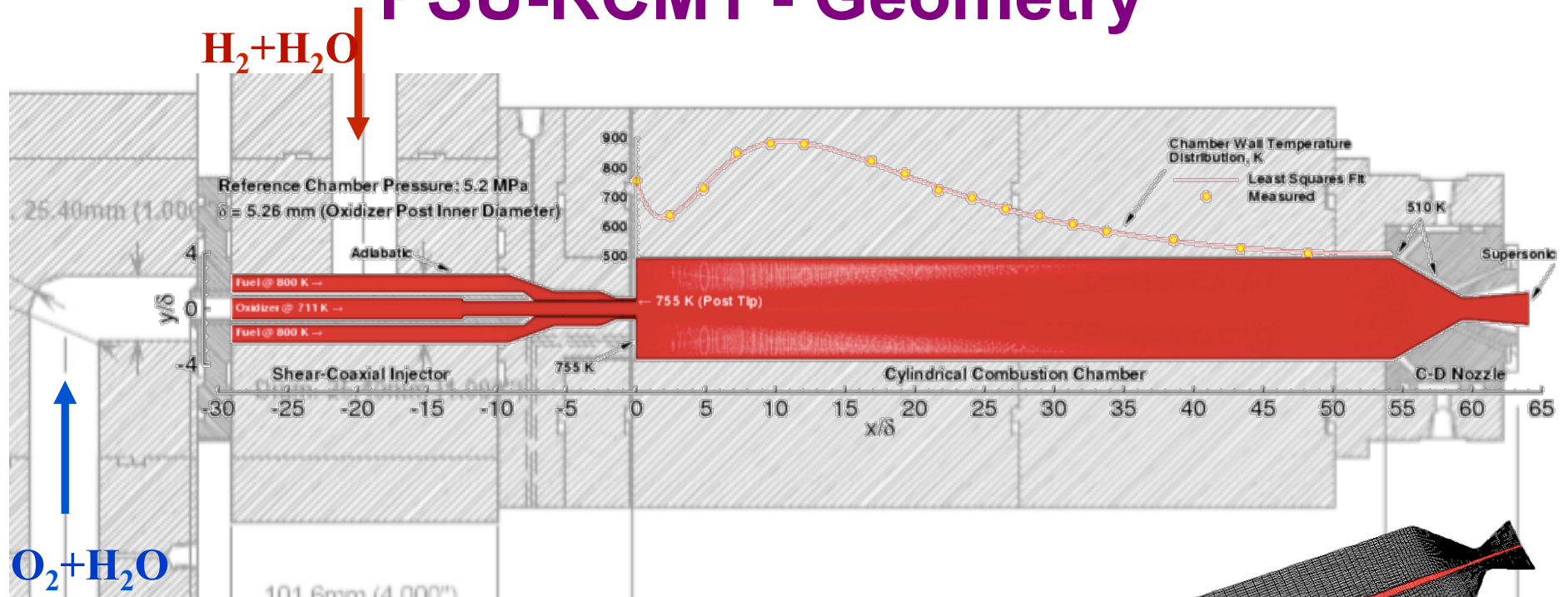


**Compressed air,
 $Z = 0.85$ pre-shock, $Z = 1.15$ post-shock**

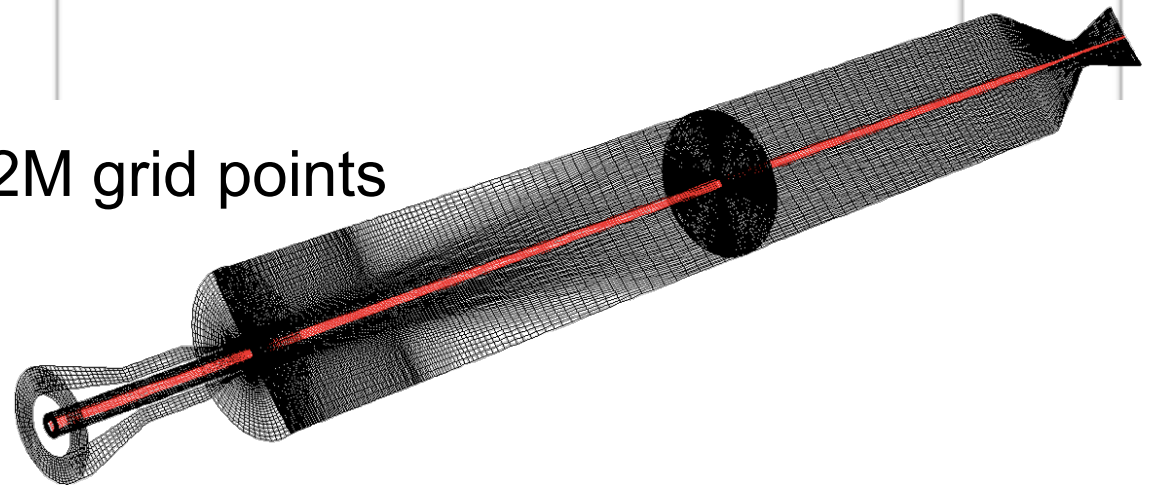
PSU RCM1

- Super-critical combustion without trans-critical event
 - Injection temperatures are high enough that real gas effects are negligible
 - Still large density gradients
 - Importance of pressure on reaction mechanism
- Simplest configuration relevant to staged combustion
 - Gas-gas H₂-O₂ shear coaxial injector
 - Cylindrical chamber instrumented for heat flux measurement

PSU-RCM1 - Geometry

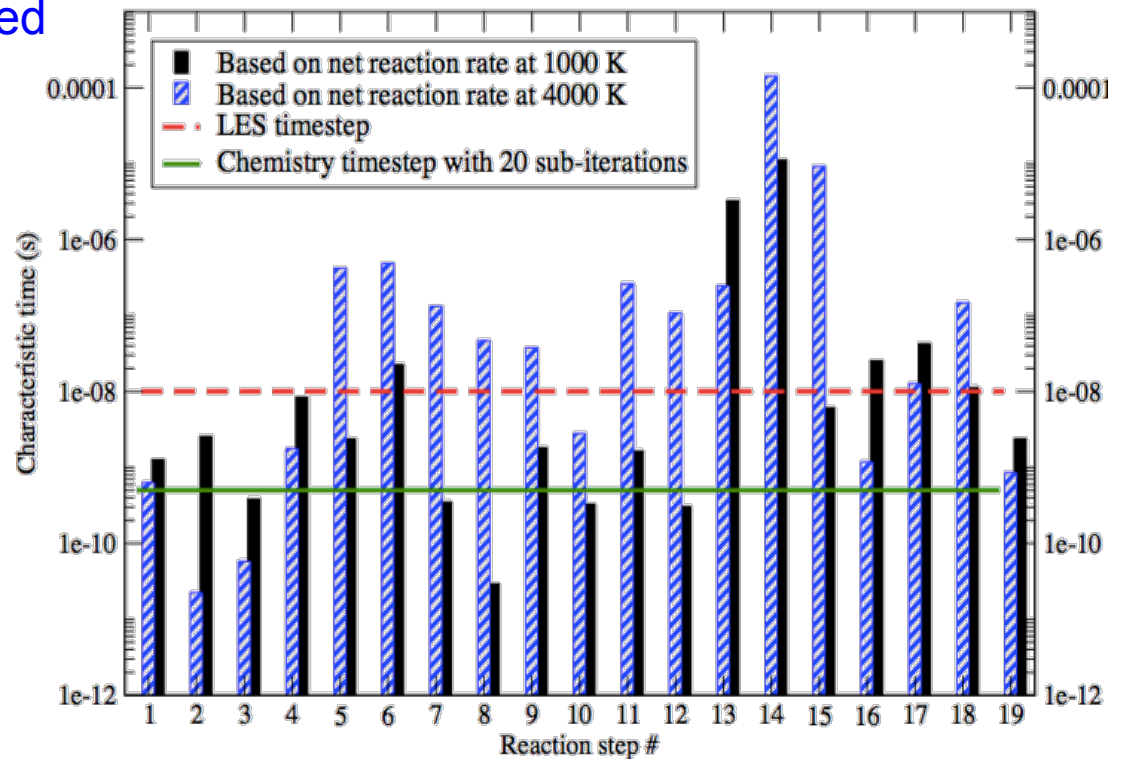


- Butterfly grid with 3.2M grid points

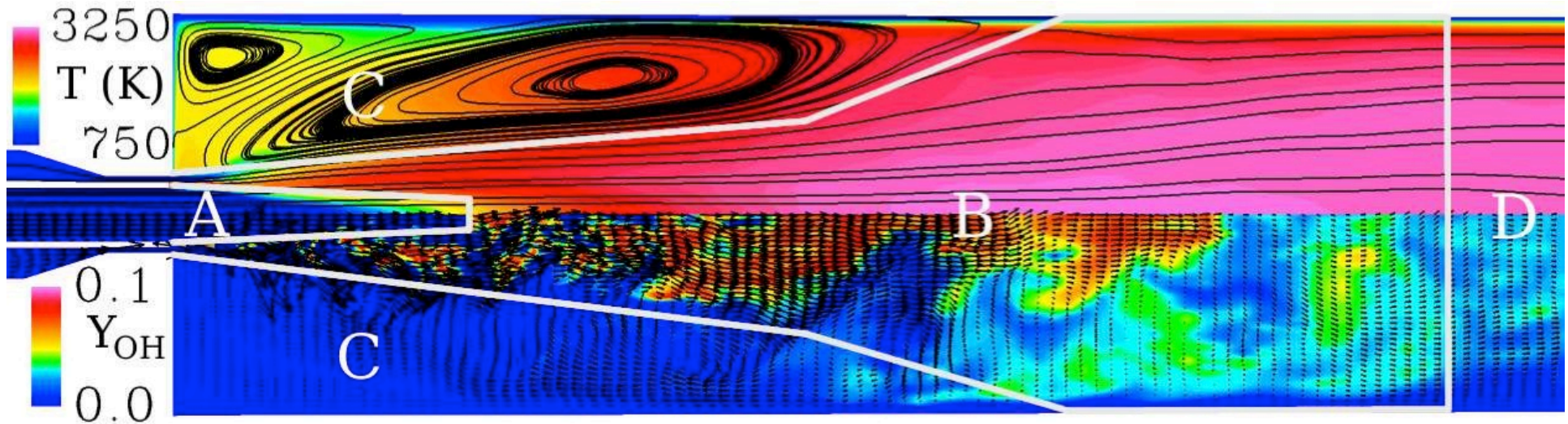


PSU RCM1 - Combustion modeling

- Characteristics of the PSU simulation
 - Good resolution in near-field and slow secondary combustion
 - Eddy Break-Up not adapted
- Detailed 21-step, 8-species mechanism (Conaire_2004)
 - Very stiff to integrate
- Simplest closure: sub-iteration scheme
- Future strategies
 - Reduced mechanism
 - LEM with ANN



PSU RCM1 - Flowfield description

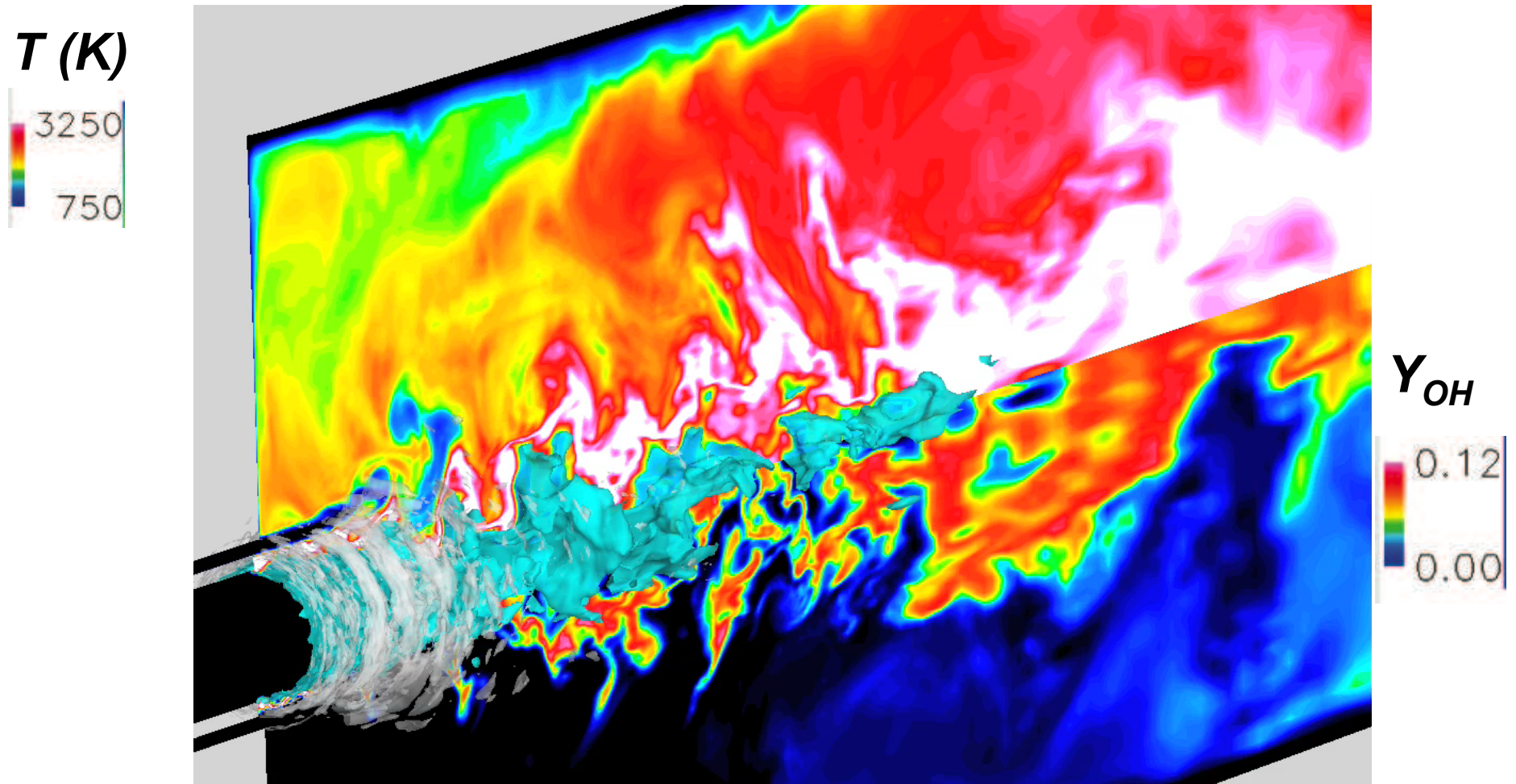


- Distinguishing 4 different zones:
 - A: oxygen jet core >>> primary diffusion flame
 - B: accelerating then decelerating flow >>> secondary combustion
 - C: recirculation zone >>> very little combustion
 - D: homogeneous flow >>> no more reaction

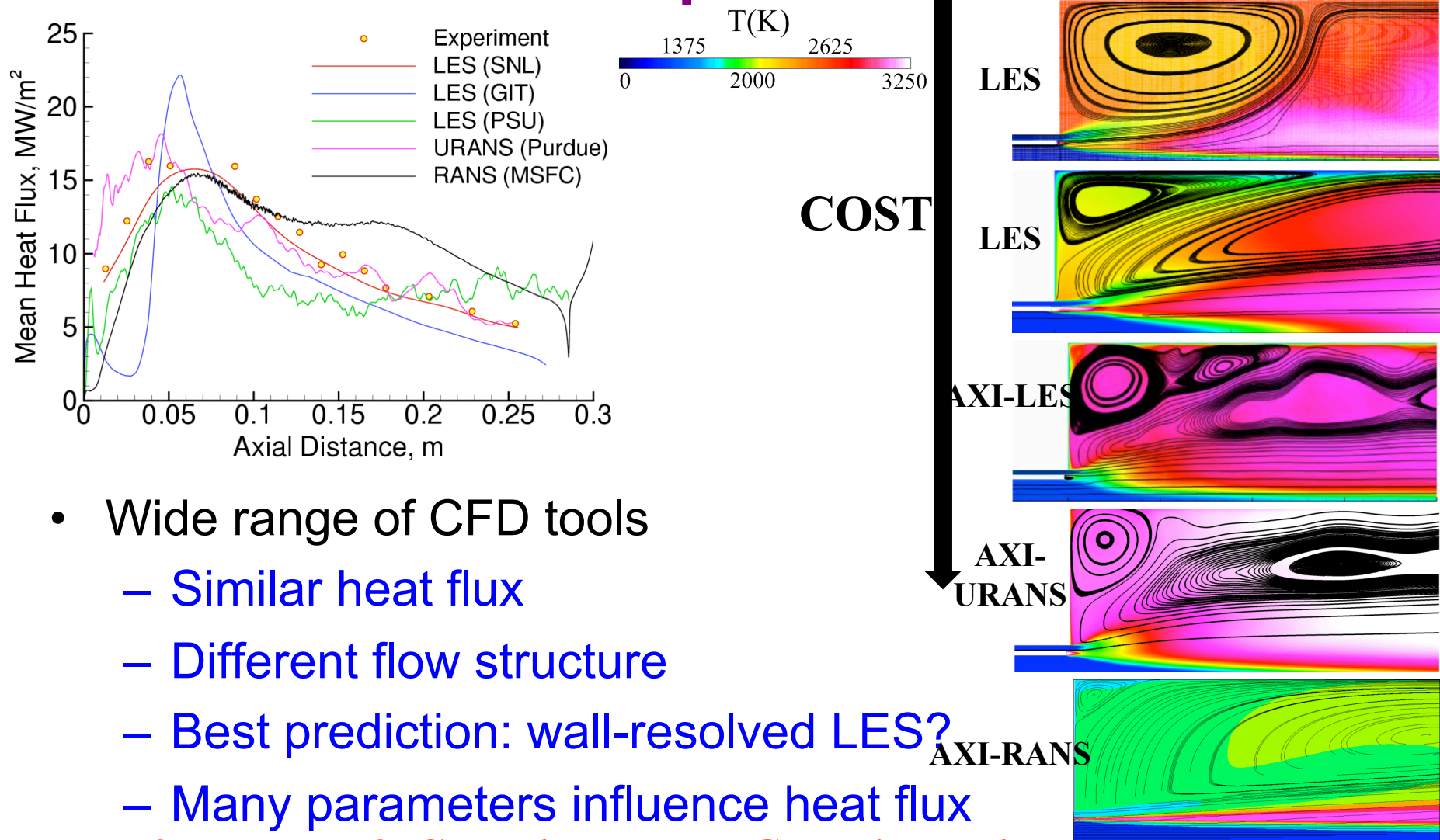
1 frame = 1 μ s

movie = 0.3 ms

Oxygen jet break-up

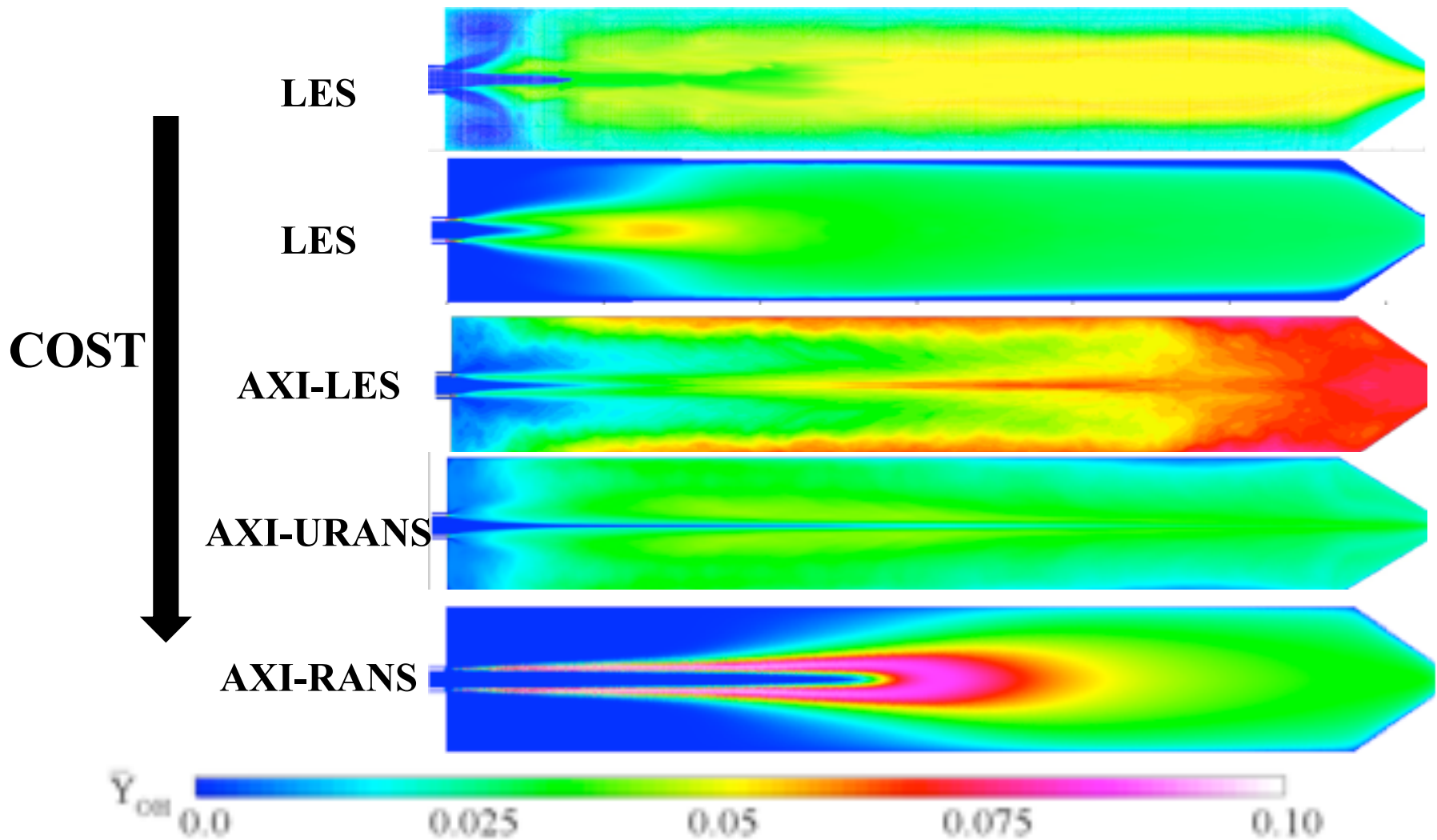


PSU RCM1 – Comparison CFD solvers

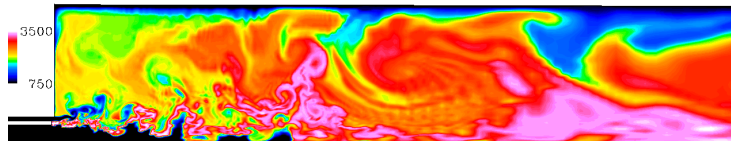


- Wide range of CFD tools
 - Similar heat flux
 - Different flow structure
 - Best prediction: wall-resolved LES?
 - Many parameters influence heat flux

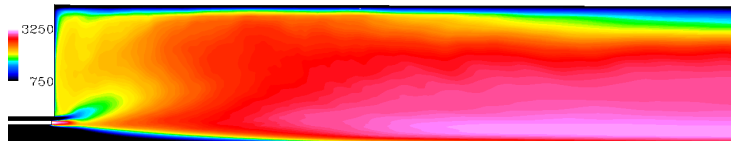
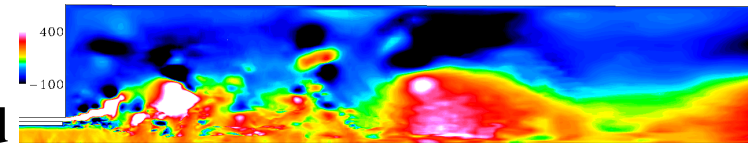
PSU RCM1 – Comparison CFD solvers



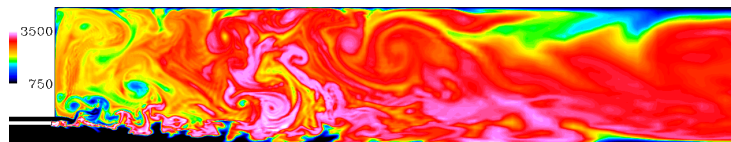
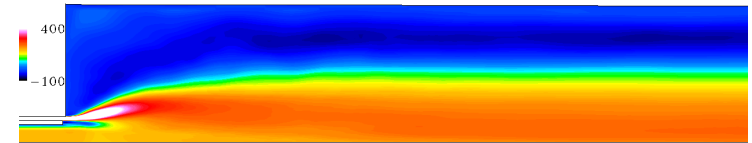
PSU RCM1 - Comparison 3D - 2D-axi LES



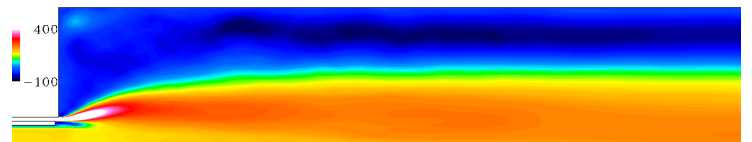
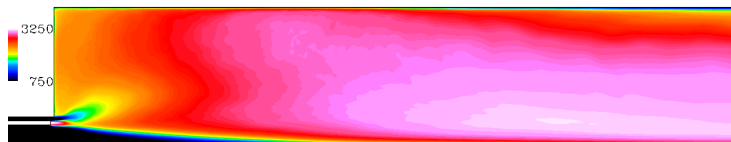
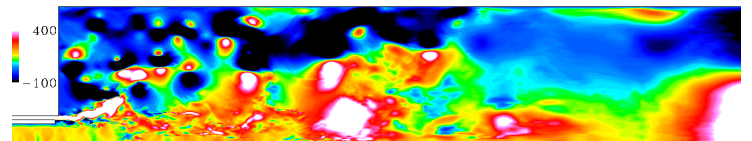
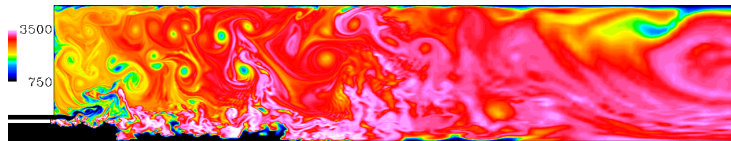
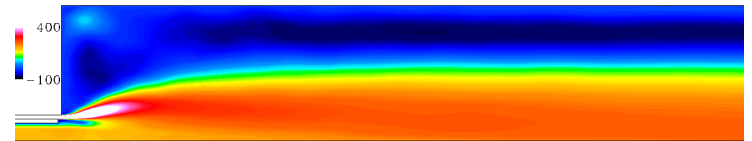
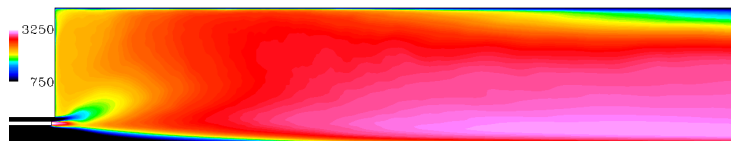
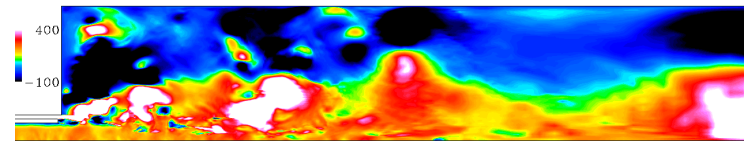
**Baseline grid
(610x94)**



**Wall-refined
grid
(610x144)**

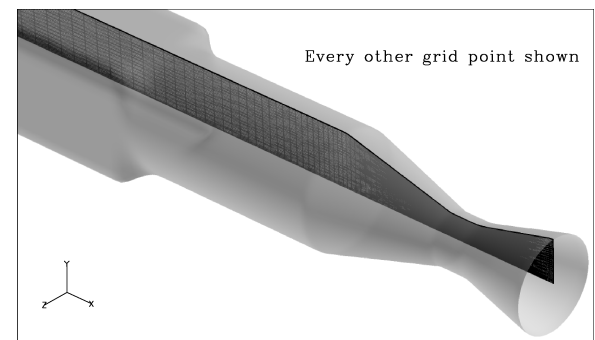
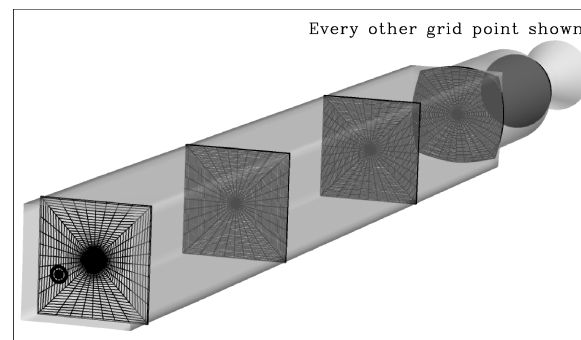
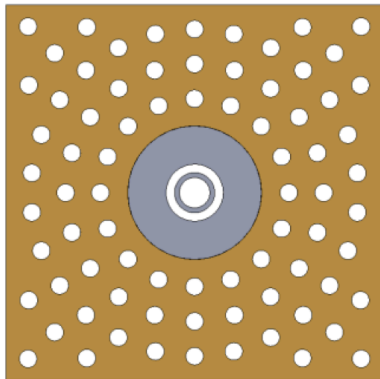


**Globally
refined grid
(916x273)**



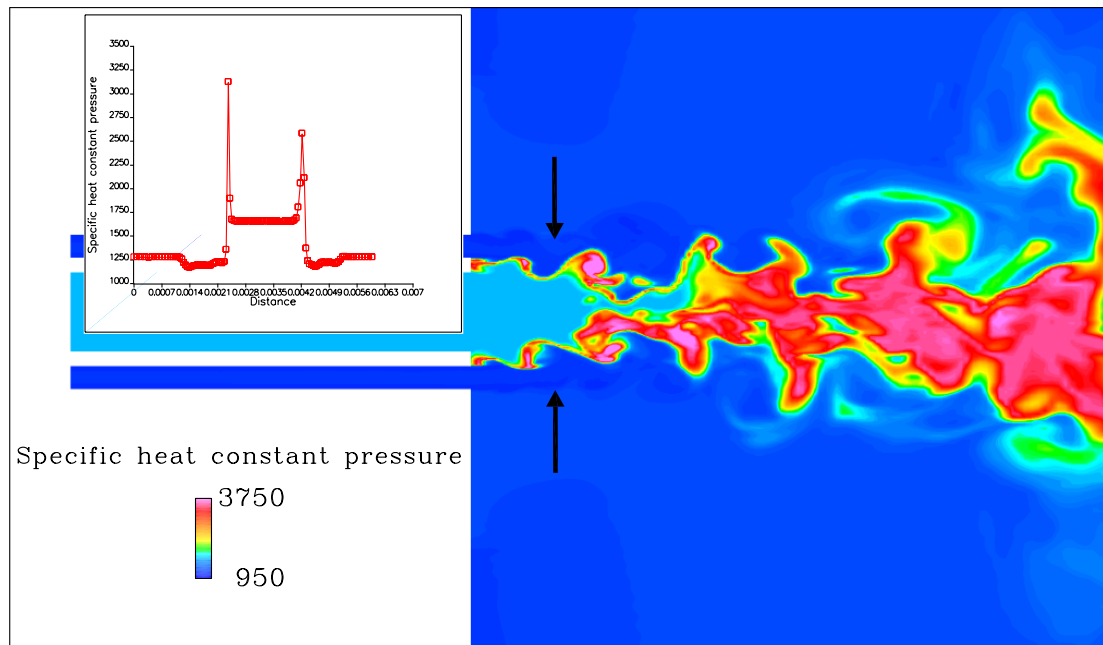
LOXGOX Experiment (PSU)

- Work on previous configuration has helped design new experimental facility
 - Square chamber for easy optical access
 - Coflow to eliminate recirculation zones
 - Perforated plate approximated as uniform flow for now



LOXGOX – Operating conditions

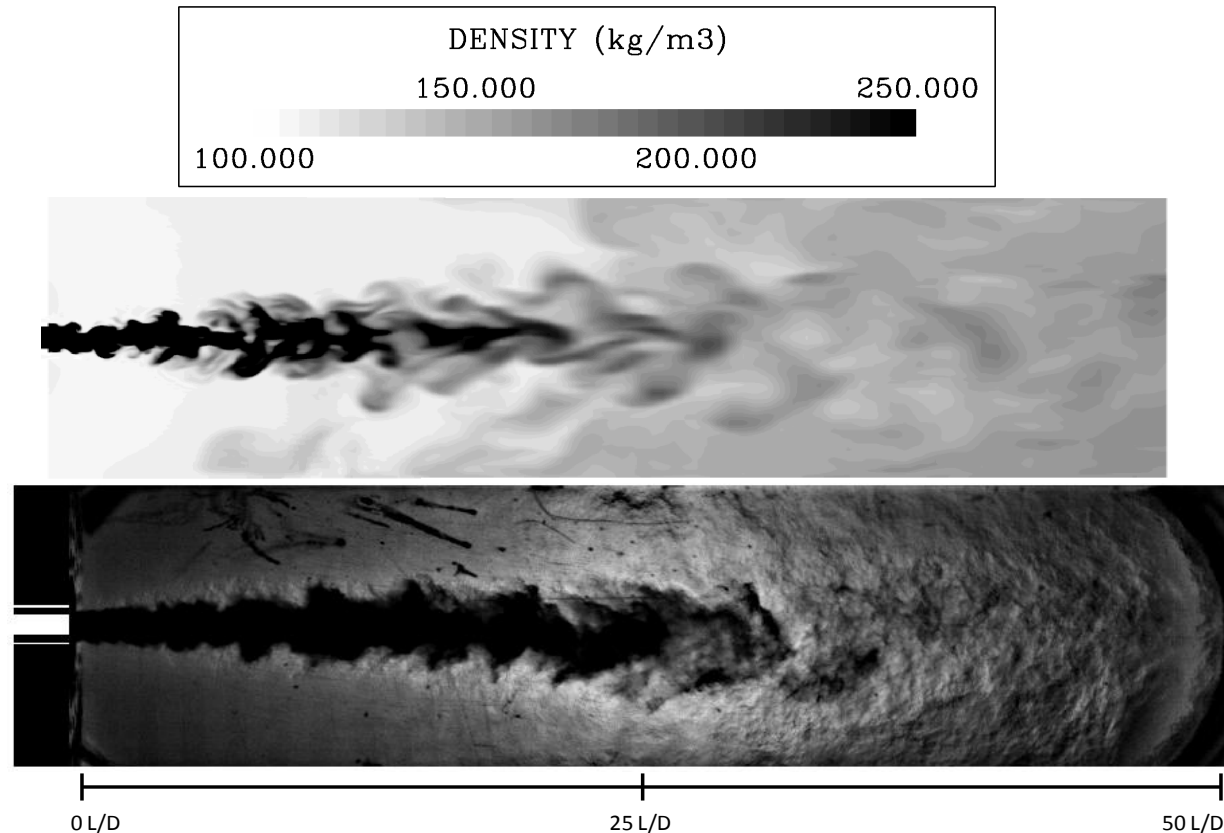
- Focus on the case with trans-critical injection
 - $P > P_c$ AND $T_{inj} < T_c$ for pure oxygen
- Hybrid scheme can capture trans-critical layer



Description	Units	Value
Main chamber		
Chamber pressure	Pa	5.750×10^6
Average density	kg.m^{-3}	138
Average velocity	m.s^{-1}	4.57
Preburner background flow		
Mass flowrate	kg.s^{-1}	0.268
Inflow density	kg.m^{-3}	84.6
Inflow temperature	K	262
Compressibility		0.998
Inflow velocity	m.s^{-1}	5.95
Injector inner post flow		
Mass flowrate	kg.s^{-1}	0.0836
Inflow density	kg.m^{-3}	1080
Inflow temperature	K	105
Compressibility		0.195
Inflow velocity	m.s^{-1}	23.3
Injector annular flow		
Mass flowrate	kg.s^{-1}	0.0557
Inflow density	kg.m^{-3}	82.3
Inflow temperature	K	269
Compressibility		0.999
Inflow velocity	m.s^{-1}	101

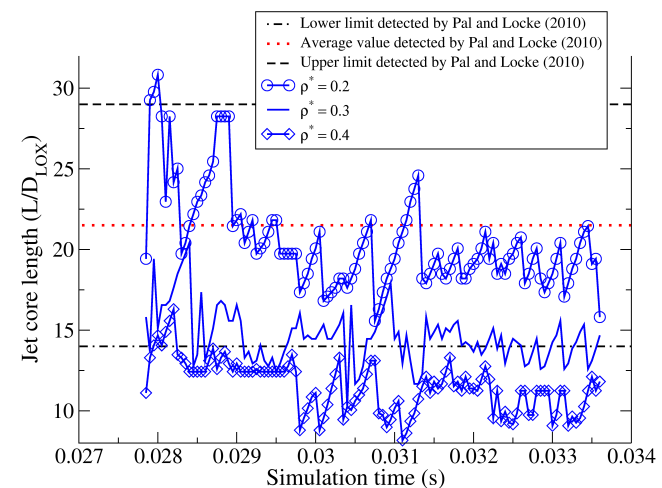
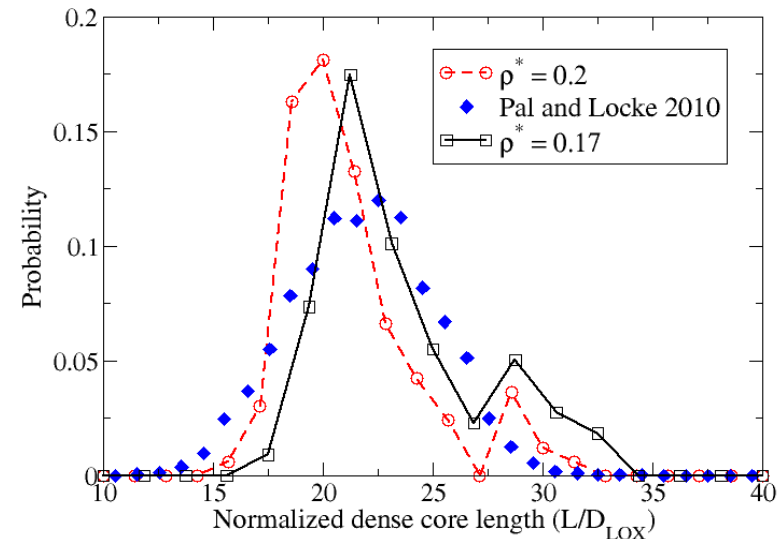
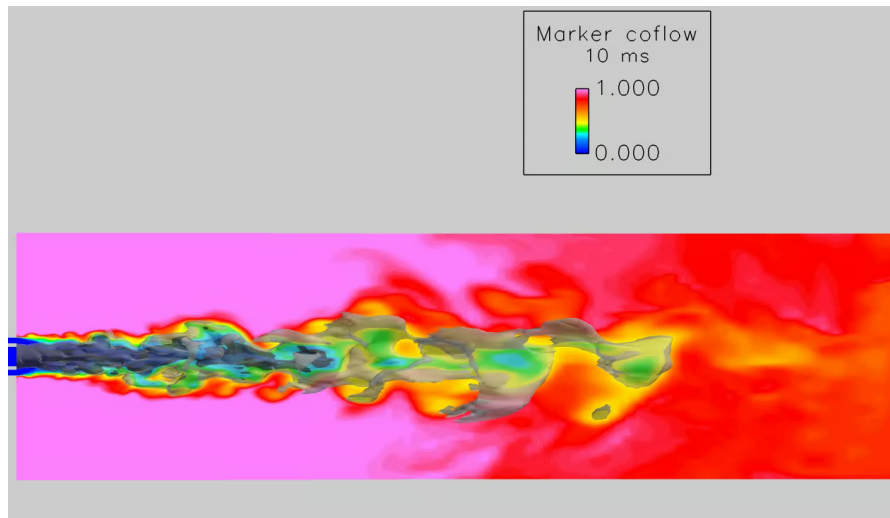
LOXGOX – Qualitative validation

- Trying to reproduce backlit images
- Good qualitative agreement



LOXGOX – Quantitative validation

- Measuring dark-core length
- Flow physics is captured with reasonable accuracy



Observations from Rocket LES

- 3D simulations are required
 - Axisymmetric cannot capture flow physics
- Complex turbulent features require LES
 - But proper subgrid closures are still needed
- Validation of single-injector flow is still difficult
 - Good experimental data is rare
- Focus should move to multi-injector flows
 - More realistic configuration

Final Summary Comments

- CFD is a tool that can be exploited with various levels of confidence and reliability for a range of problems
- Sometimes asking too much of a simple and reliable model may not be the proper thing to do....
- Key areas to be aware of
 - Numerical scheme' s strengths and limitations
 - Choice of grid and boundary conditions
 - Turbulence closures (RANS, URANS, DES or LES)
 - Scalar mixing closure (turbulent and molecular)
 - Reaction kinetics closure (finite-rate, mixture fraction)
 - Parallel optimization and scalability is essential

Further Reading

- All models and results discussed are in published papers
 - Cited work papers are available upon request
- Many excellent reviews and books are also available
 - Poinso & Veynante: Theoretical and Numerical Combustion, Edwards, 2nd
 - Reviews by Pitsch (Ann. Rev. 2006), Janicka (Symp 2006), Peters (2008), Candel etc...
- Other papers are available
- LEM stand-alone codes can be used to learn and in needed implemented into in-house codes